

Structur I

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SMW

Birzeit University
Department of Civil Engineering
ENCE333 Structural Analysis I
First Exam

Instructor: Dr. Mirvat Bulbul

16 October, 2010

Name: ~~XXXXXXXXXX~~

ميرفات بولبول

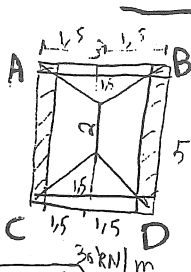
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Question 1 (15 marks)

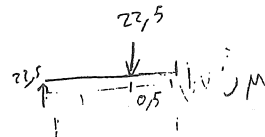
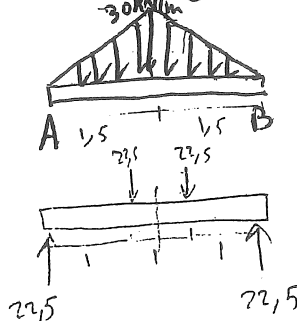
15

A rectangular two-way slab is 3m by 5m in plan dimensions. It is supported on beams on all 4 sides. The slab has a uniformly distributed load of 20kN/m².

Find the loading distribution on the short and long beams and draw in each case the corresponding shear and bending moment diagrams and indicate key values.



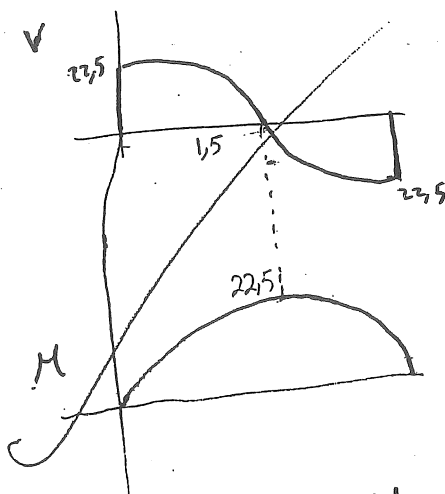
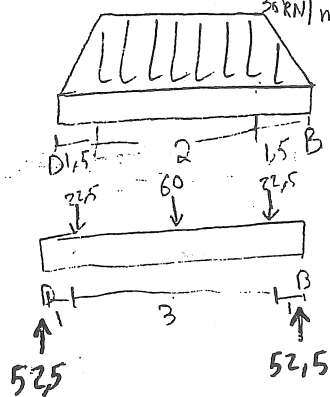
the intensity = $20 \times 1,5 = 30 \text{ kN/m}$



~~22,5 - 22,5~~

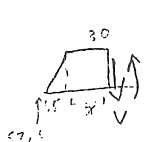
$$-22,5 \times (1,5) + 22,5(0,5) + M = 0$$

$$M = \frac{3,75}{2} = 22,5$$



Beam ~~AB~~ CD liker beam AB
in shear & bending moment diagram

Beam AC liker beam DB
in shear & bending moment diagram



$$52,5 - 22,5 - 30x = 0$$

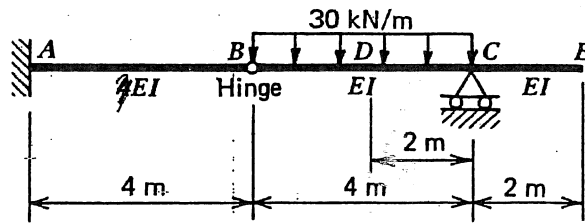
$$x = 1$$

$$M - 52,5 \times 2,5 + 22,5(1,5) + 30(0,5) + M = 0$$

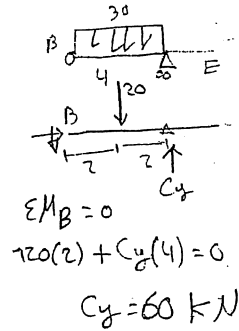
$$M = 82,5$$

Question 2 (30 marks) 10

Using the moment area theorems, find the deflection and rotation at **E** for the beam shown below.

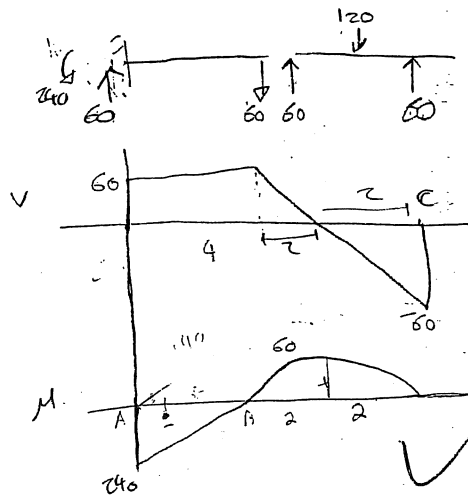


$E = 200 \text{ GPa}; I = 50 \times 10^6 \text{ mm}^4$

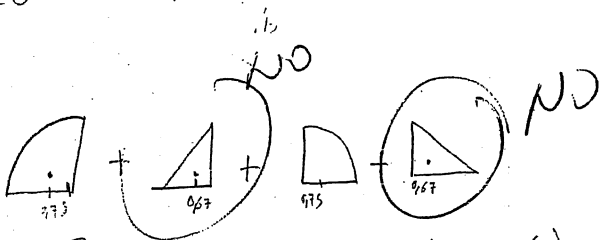


$A_{\text{parabola}} = \frac{2}{3}bh$
 $\bar{x} = \frac{3}{8}b$
 $\theta_{CL} = \theta_{CR}$
 $y_C = 0$
 $\theta_A = 0$
 $y_A = 0$

$t_{B/A} = \delta_B = \int \frac{M}{EI} dx$
 $= \frac{8}{3} \left(\frac{240 \times 4}{2} \right) \times \frac{1}{EI}$
 $= \frac{320}{EI} = \frac{320}{200 \times 10^9 \times 50 \times 10^{-8} \times 10^{-12}}$
 $= 3,2 \times 10^{-5} \text{ m}$
 $0,032 \text{ m}$
 $= 3,2 \text{ mm}$
 $0,128 \text{ mm}$



$\frac{120}{4} = \frac{60}{x}$
 $30x = 60$
 $x = 2$



$t_{C/B} = \frac{1}{EI} \left[2,75 \times \left(\frac{2}{3} \times 2 \times 60 \right) + 2,67 \times \left(\frac{1}{2} \times 2 \times 60 \right) + 1,25 \times \left(\frac{2}{3} \times 2 \times 60 \right) + 1,33 \times \left(\frac{1}{2} \times 2 \times 60 \right) \right] = \frac{560}{EI} = 5,6 \times 10^{-5} \text{ m}$

from similar triangles:

$\theta_{CL} - \theta_A = \text{the area}$
 $\theta_{CL} = -4 \times \frac{240}{2(4EI)} + \left[\frac{2}{3} \times 2 \times 60 + \frac{1}{2} \times 2 \times 60 + \frac{2}{3} \times 2 \times 60 + \frac{1}{2} \times 2 \times 60 \right] \times \frac{1}{EI}$
 $= \frac{-120}{EI} + \frac{280}{EI} = \frac{160}{EI} = 1,6 \times 10^{-5} \text{ rad}$

$\theta_{CL} = \theta_{CR} = \theta_E$

$\theta_{CL} = \frac{\delta_E}{2} \Rightarrow \delta_E = \theta_{CL} \times 2 = 3,2 \times 10^{-5} \text{ m}$

Discontinuity

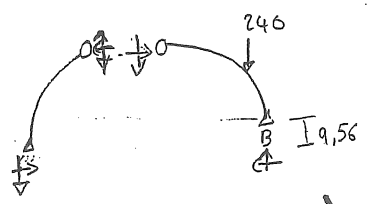
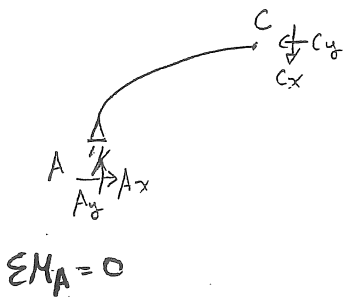
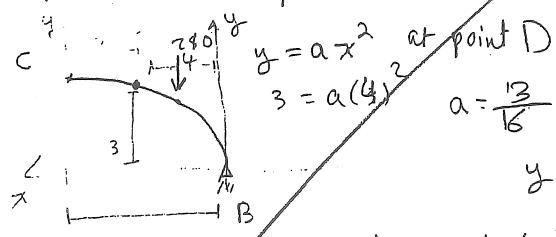
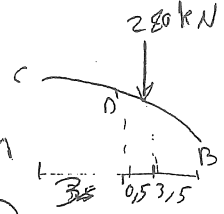
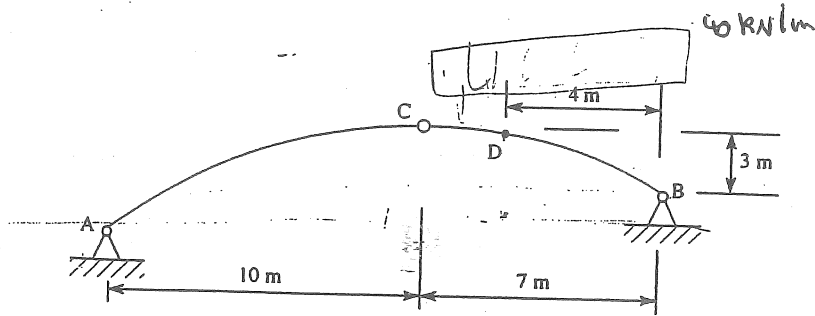
NO

OK

Question 3 (30 marks)

12

The three-pinned arch has the shape of a parabola with its origin at C. The arch carries a uniform horizontally distributed load of intensity 40 kN/m over the part CB only. Calculate the internal forces at D (axial and shear forces and bending moment).

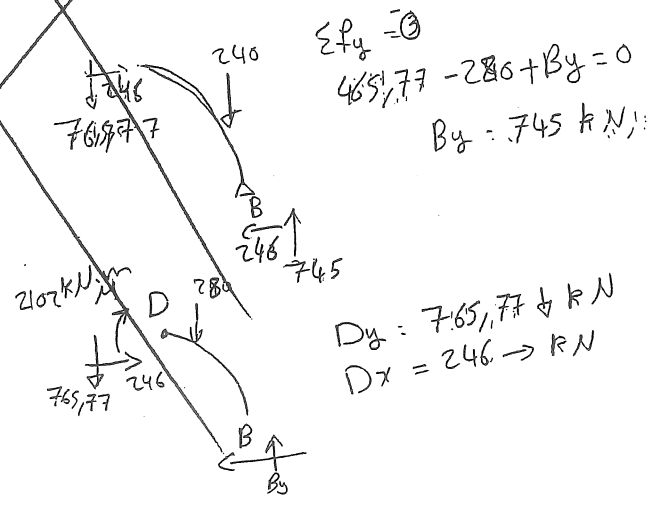


$\Sigma M_B = 0$
 $C_y(7) - C_x(9,18) + 280(3,5) = 0$
 $7C_y - 9,18C_x = -980 \dots (1)$

$\Sigma M_A = 0$
 $-10C_y + 18,75(C_x) = 0$
 $18,75C_x = 10C_y$
 $C_y = 1,875C_x \dots (2)$

(1) $7(1,875C_x) - 9,18C_x = -980$
 $3,945C_x = -890$
 $C_x = 248 \text{ kN}$
 $C_y = 465,777 \text{ kN}$

$y = ax^2$
 $= \frac{3}{16} \times 10^2 = \frac{3}{16} \times 100 = 18,75$



$\Sigma F_y = 0$
 $465,777 - 280 + B_y = 0$
 $B_y = 745 \text{ kN}$

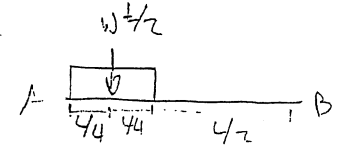
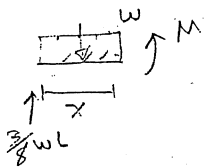
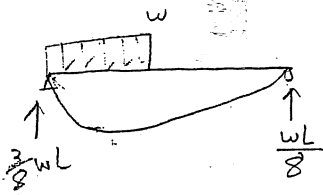
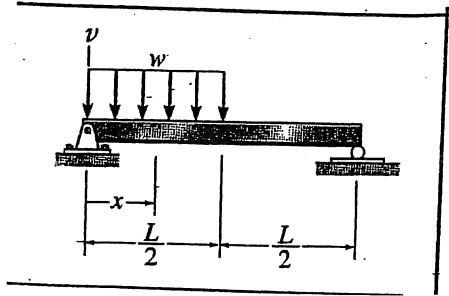
$D_y = 765,777 \text{ kN}$
 $D_x = 246 \text{ kN}$

$\Sigma M_D = 0$
 $745 \times 4 - 246 \times 3 - 280(0,5) - M = 0$
 $M = 2102 \text{ kN.m}$

خلف الورقة

Question 4 (25 marks)

Derive the elastic curve profile for the loaded beam below. Assume constant flexural rigidity, EI.



$$\sum M_A = 0$$

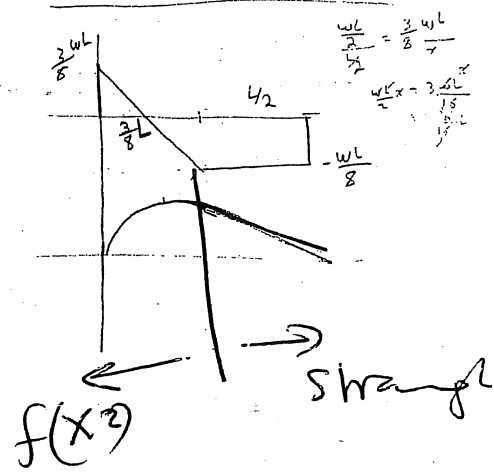
$$-\frac{wL}{2} \cdot \frac{L}{4} + B \cdot L = 0$$

$$+\frac{wL^2}{8} = B \cdot L$$

$$B = \frac{wL}{8}$$

$$\frac{wL}{2} - \frac{wL}{8} = f \cdot L$$

$$\frac{3}{8} wL = A \cdot L$$



$$\sum M = 0$$

$$-\frac{3}{8} wL \cdot x + \frac{w x^2}{2} + M = 0$$

$$M = \frac{3}{8} wLx - \frac{w x^2}{2}$$

$$\frac{d^2 y}{dx^2} = \frac{M}{EI} = \frac{1}{EI} \left(\frac{3}{8} wLx - \frac{w x^2}{2} \right)$$

$$\frac{dy}{dx} = \frac{1}{EI} \left(\frac{3}{16} wLx^2 - \frac{w x^3}{6} + A \right)$$

$$y = \frac{1}{EI} \left(\frac{3}{48} wLx^3 - \frac{w x^4}{24} + Ax + B \right)$$

At $x=0$ $y=0 \Rightarrow B=0$

At $x=L$ $y=0$

$$0 = \frac{1}{EI} \left(\frac{3}{48} wL^4 - \frac{wL^4}{24} + AL \right)$$

$$AL = \frac{-3}{48} wL^4 + \frac{wL^4}{24}$$

$$A = \frac{-3}{48} wL^3 + \frac{wL^3}{24} = \frac{-wL^3}{48}$$

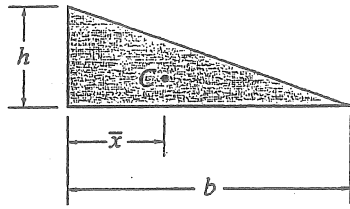
$$y = \frac{1}{EI} \left(\frac{3}{48} wLx^3 - \frac{w x^4}{24} - \frac{wL^3 x}{48} \right)$$

7

No 2nd func

2 functions for integ.

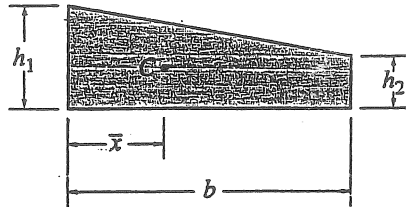
Geometric Properties of Areas



$$A = \frac{1}{2}bh$$

$$\bar{x} = \frac{1}{3}b$$

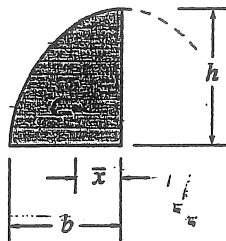
Triangle



$$A = \frac{1}{2}b(h_1 + h_2)$$

$$\bar{x} = \frac{b(2h_2 + h_1)}{3(h_1 + h_2)}$$

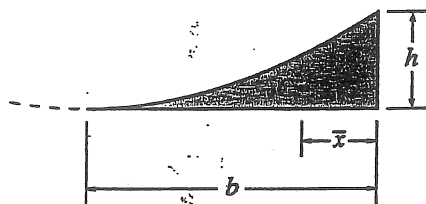
Trapezoid



$$A = \frac{2}{3}bh$$

$$\bar{x} = \frac{3}{8}b$$

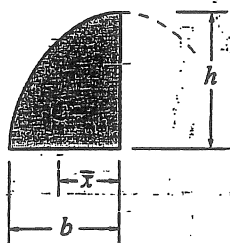
Semi Parabola



$$A = \frac{1}{3}bh$$

$$\bar{x} = \frac{1}{4}b$$

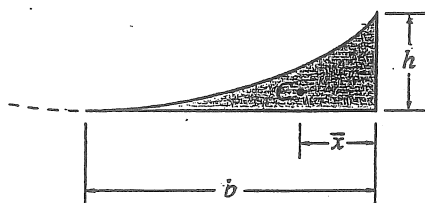
Parabolic spandrel



$$A = bh \left(\frac{n}{n+1} \right)$$

$$\bar{x} = \frac{b(n+1)}{2(n+2)}$$

Semi-segment of n th degree curve



$$A = bh \left(\frac{1}{n+1} \right)$$

$$\bar{x} = \frac{b}{(n+2)}$$

Spandrel of n th degree curve

FIRST Mirvat Bulbul
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90
100

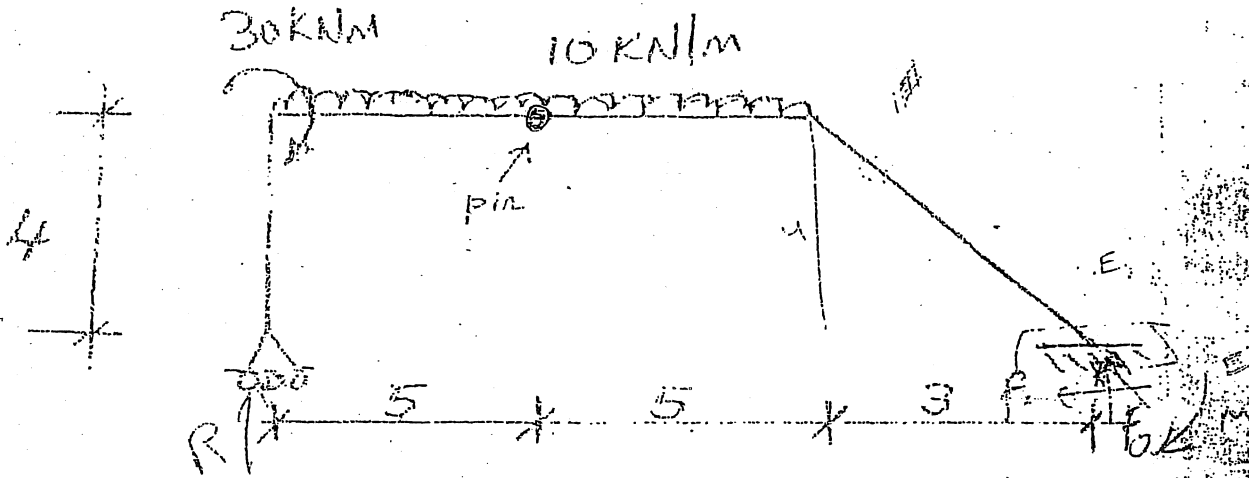
BirZeit University
Civil Engineering Department
Structural Analysis I CE333
First Exam

Date: 10/12/2000

Dr. Mirvat Bulbul
Dr. Elias Sagan

Question 1

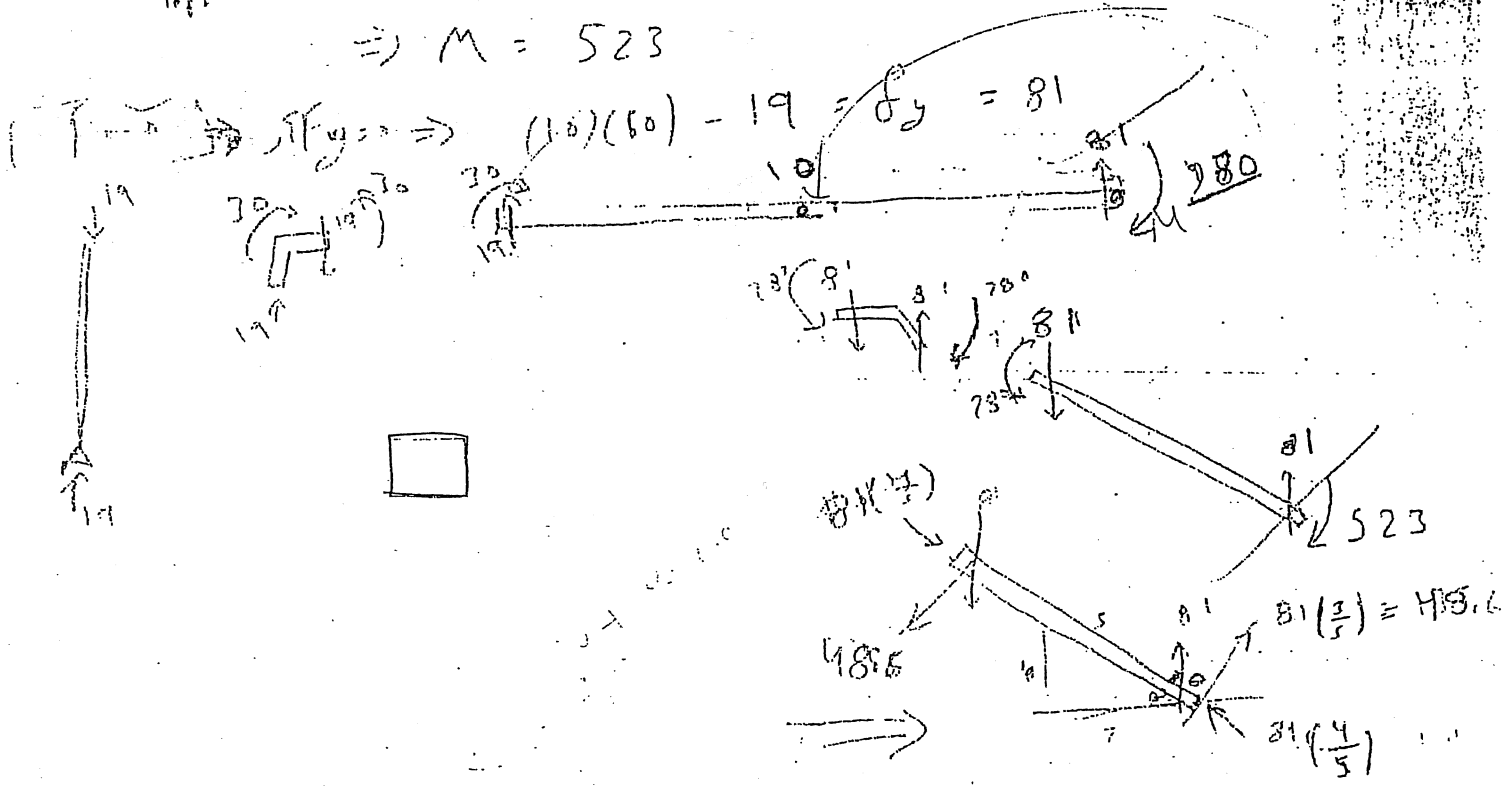
For the following framed structure, draw the shear force and bending moment diagrams, giving principal values.



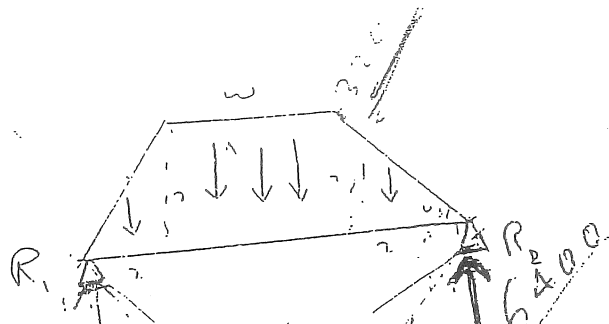
$$M_F = M \Rightarrow (10)(10)(81) - 30 - 17R = M$$

$$M_{pin} \Rightarrow (10)(5)(2.5) - 30 = 5R \Rightarrow R = 19$$

$$\Rightarrow M = 523$$

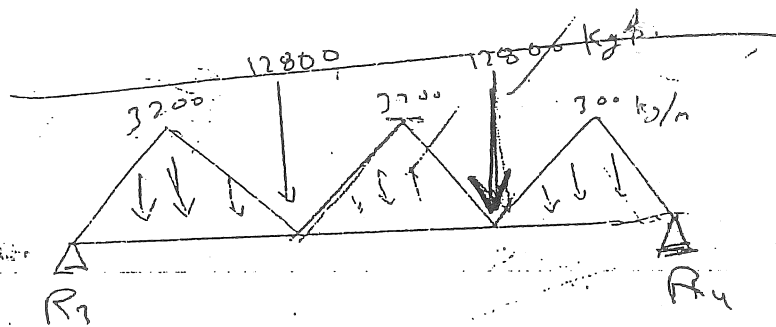


(b)



$$w = (800)(4) = 3200 \text{ kg/m}$$

$$R_1 = R_2 = \frac{[(3200)(2) + 2(\frac{1}{2})(12)(3200)]}{2} = 6400 \text{ kg}$$



$$R_3 = R_4 = \frac{[(12800)(2) + 6(\frac{1}{2})(12)(3200)]}{2} = 22400 \text{ kg}$$

(c) Column B6 in the second floor carries the loads of that floor ~~and~~ in addition to the loads of three floors above it.

The load ~~on~~ on one floor = $(8)(4)(800) = 192000 \text{ kg}$

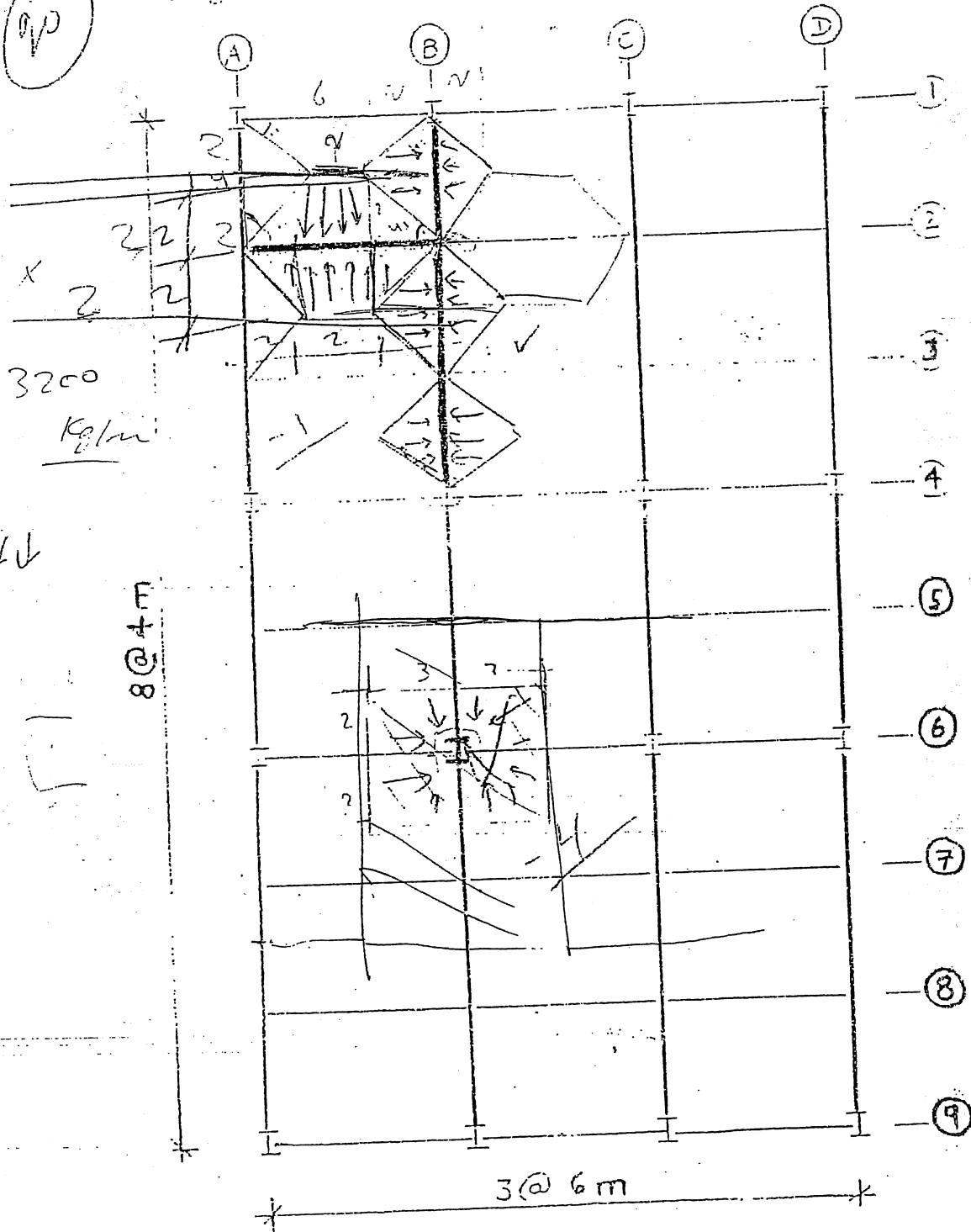
The load on the column = the load on four floors

$$= (4)(192000)$$

$$= 768000 \text{ kg}$$

Problem 4 : The sketch below is a plan of a typical floor in a 5-storey building. The floor is to be designed to support a total service load of 800 kg/m^2 .

- On the plan draw clearly and to scale the shape of the loads to be supported by the elements indicated by the heavy lines (Beam AB on axis 2, Girder 1-4 on Axis B, and Column B6).
- Draw Beam AB on axis 2 and Girder 1-4 on axis B with the loads they support as free bodies and determine the values of the loads and reactions.
- Determine the value of the axial load on Column B6 in the second floor (first after the ground).



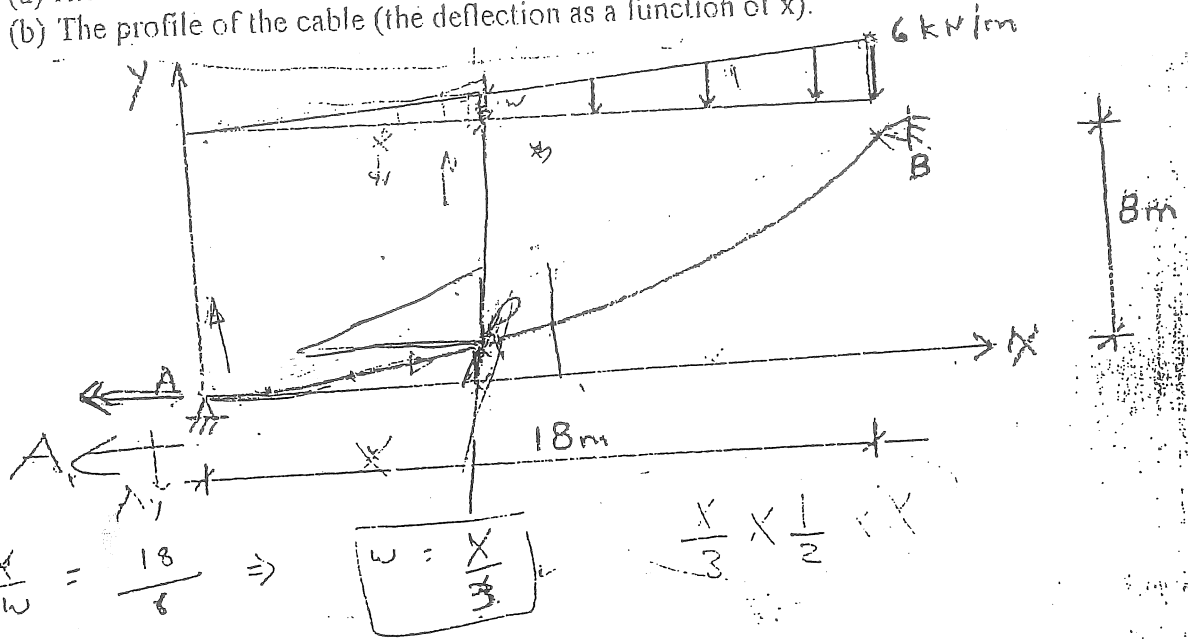
kg/m^2

Problem 3 : The cable shown has a horizontal slope at support A. For the cable and loading shown, determine:

(a) The tension in the cable as a function of x .

(b) The profile of the cable (the deflection as a function of x).

25



$$\frac{x}{w} = \frac{18}{8} \Rightarrow w = \frac{x}{\frac{8}{3}} = \frac{x}{3} \times \frac{1}{2} = \frac{x}{6}$$

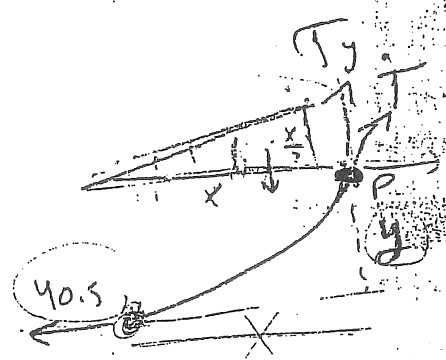
$$\sum M_A = 0 \Rightarrow \frac{(6)(18)(9)}{2} = A_y(18)$$

$$A_y = 40.5 \checkmark$$

$$A_x = 0$$

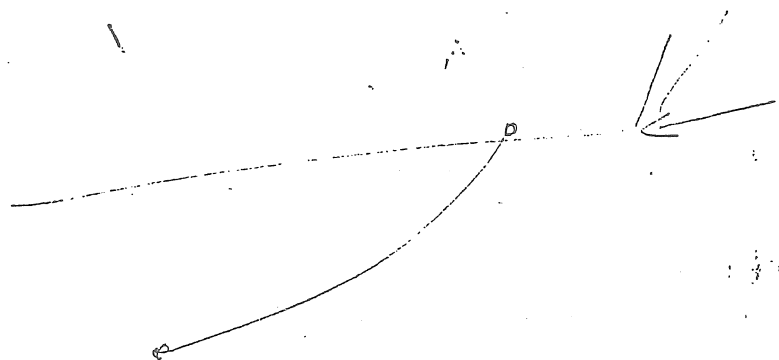
$$T_x = 40.5 \quad T_y = \frac{1}{2}(x)\left(\frac{x}{6}\right) = \frac{x^2}{6}$$

$$T = \sqrt{(40.5)^2 + \left(\frac{x^2}{6}\right)^2}$$



$$\sum M_P = 0 \Rightarrow (40.5)(y) = \frac{1}{2}\left(\frac{x}{3}\right)\left(\frac{x}{3}\right) = \frac{x^3}{18}$$

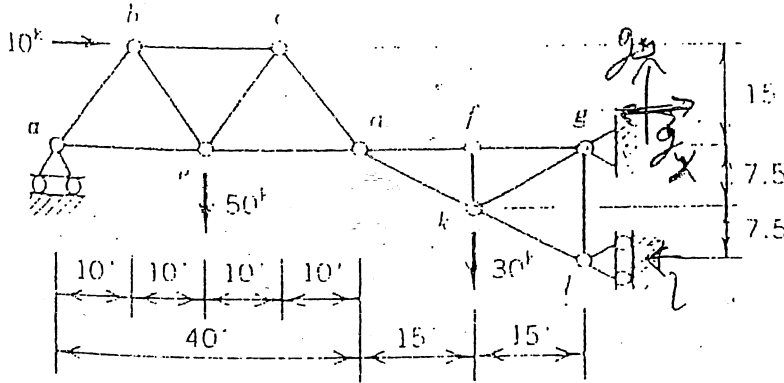
$$\Rightarrow y = \frac{x^3}{729}$$



Question 2

20

For the loading configuration shown, find the axial force in member gk .



$$\sum M_a = 0 \Rightarrow 10(15) + (50)(20) + 130(55) = g_y(70) - L(15)$$

$$70g_y - 15L = 2800 \quad \text{--- (1)}$$

$M_d = 0$ \Rightarrow $130(15) = g_y(30) - L(15)$

$$30g_y - 15L = 450 \quad \text{--- (2)}$$

$$40g_y = 2350 \Rightarrow g_y = 58.75 \checkmark$$

$$\Rightarrow L = \frac{450 - 30(58.75)}{-15} = 87.5 \checkmark$$

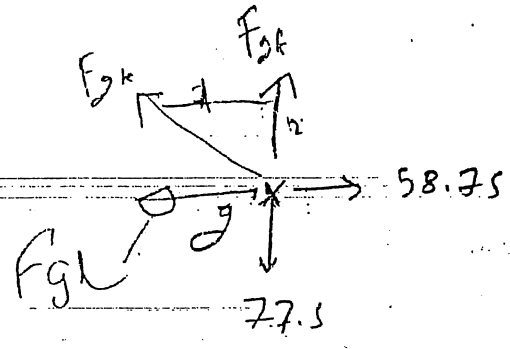
$$\sum F_x = 0 \Rightarrow g_x = 87.5 - 10 = 77.5 \checkmark$$

$$\sum F_v = 0$$

$$58.75 = F_{gk} \left(\frac{1}{\sqrt{5}} \right)$$

$$\Rightarrow F_{gk} = 131.36 \text{ k}$$

5



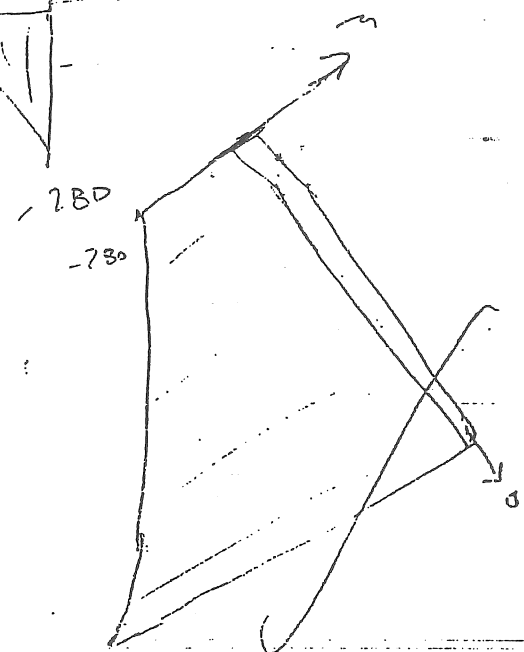
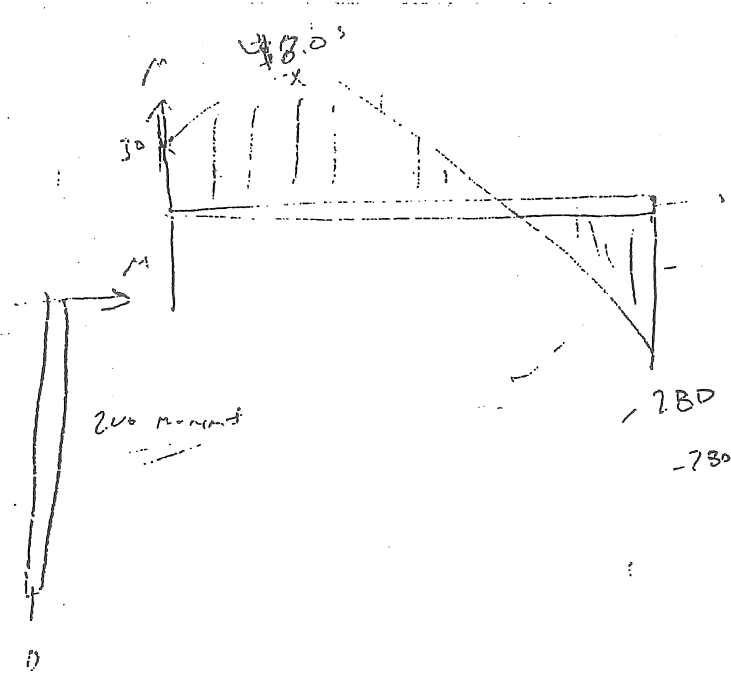
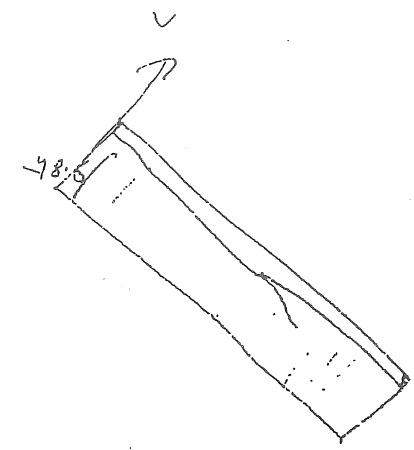
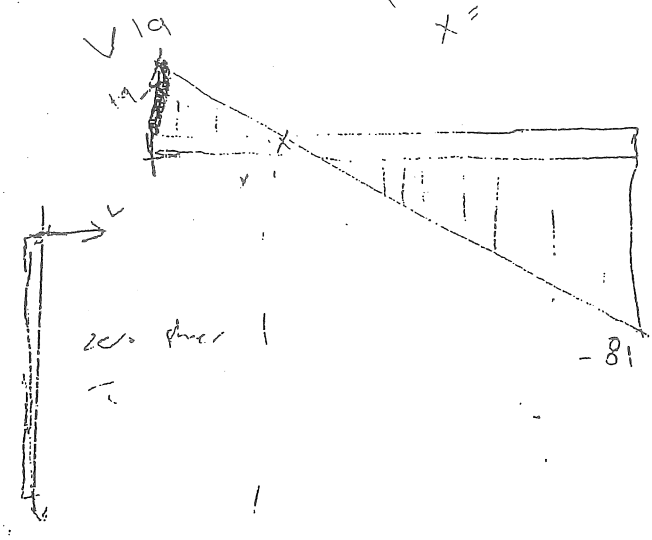
$$\frac{19}{x} = \frac{3}{10-x}$$

$$19(10-x) = 3x$$

$$190 - 19x = 3x$$

$$190 = 22x$$

$$x = 8.64$$



523.

frachten
res. neue

Handwritten notes at the top of the page.

Handwritten word "Solve" in the top right corner.

Name: _____

Handwritten word "Key" written across the name line.

Birzeit University
Faculty of Engineering
Department of Civil Engineering

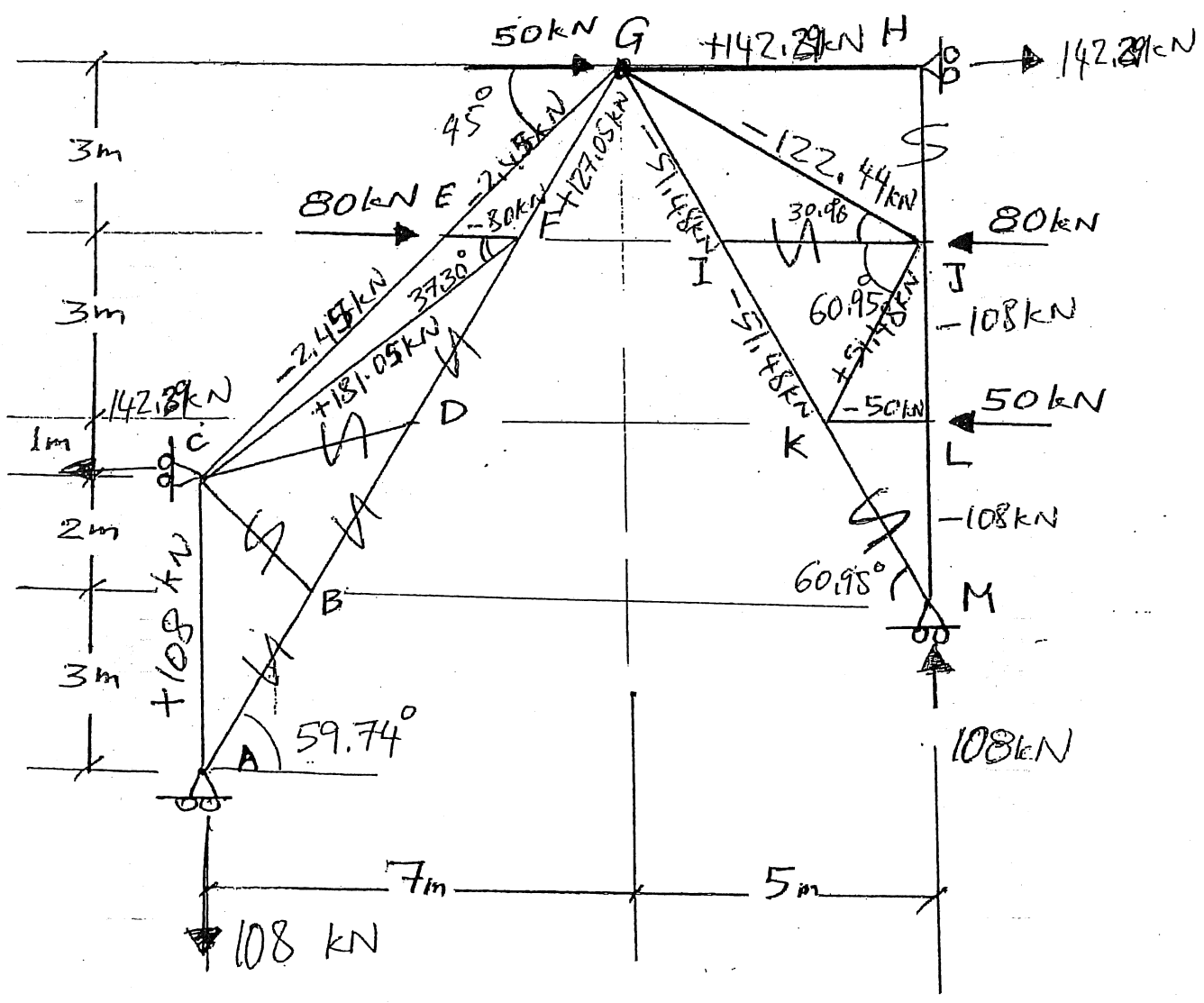
Handwritten text "First" with a horizontal line underneath.

Structural Analysis I, ENCE 333

Quiz

Tuesday, June 12, 2012

(20 points) Determine the zero-force members, the reactions, and the force in each member. Summarize the results on the truss using a "+" sign for tension.



Key

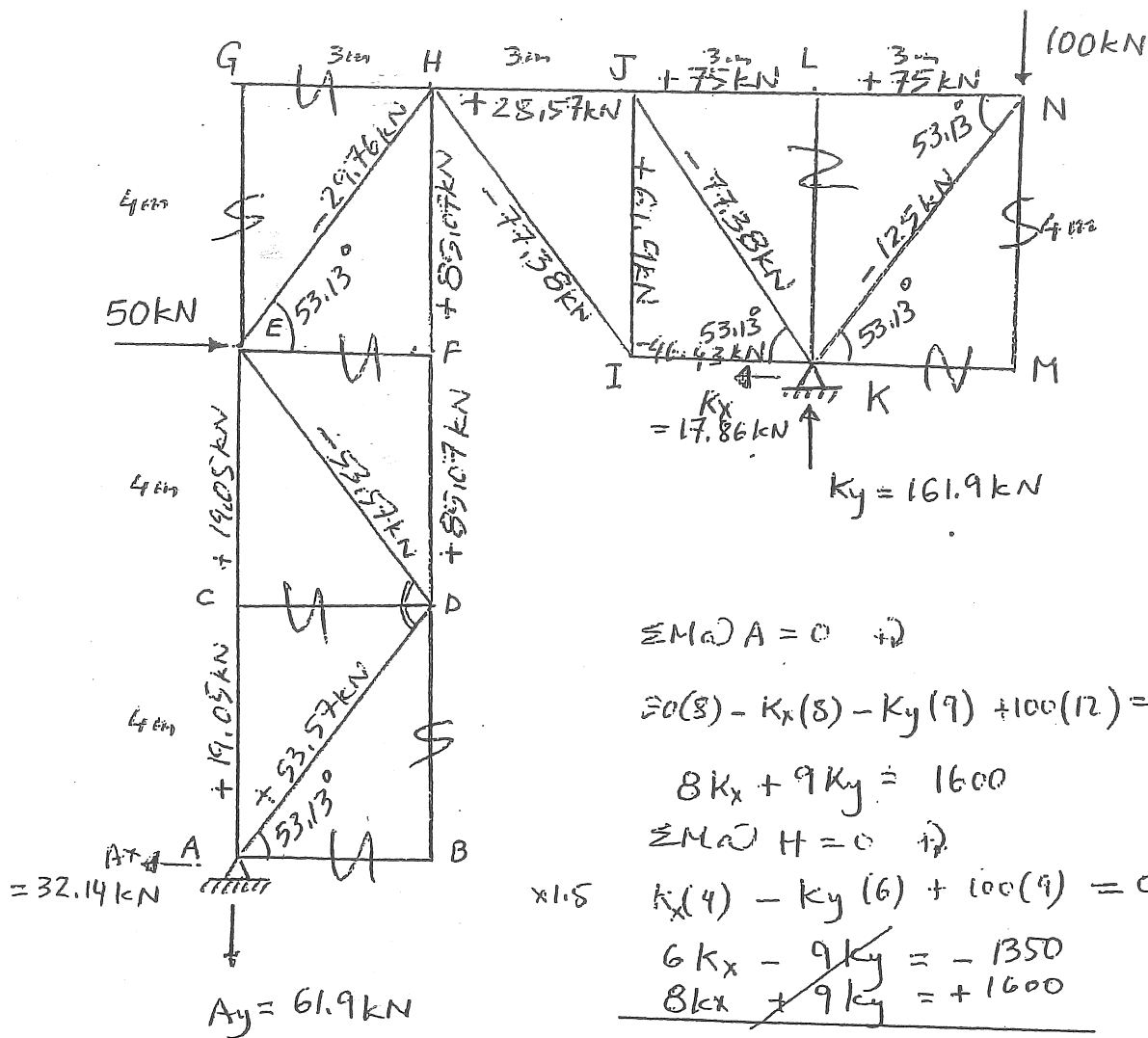
ENCE333, Structural Analysis I

First Exam

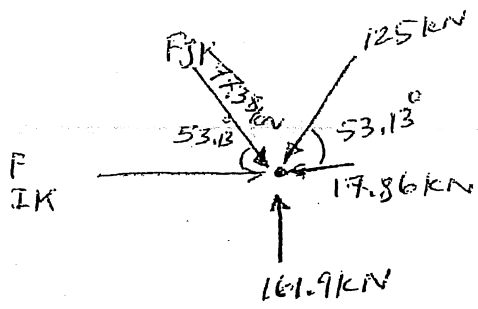
Saturday, June 2, 2012

1. Identify the zero-force members then determine the force in each member.

a) (8 points)



Take joint K:



$$\sum F_y = 0 \uparrow \quad (2)$$

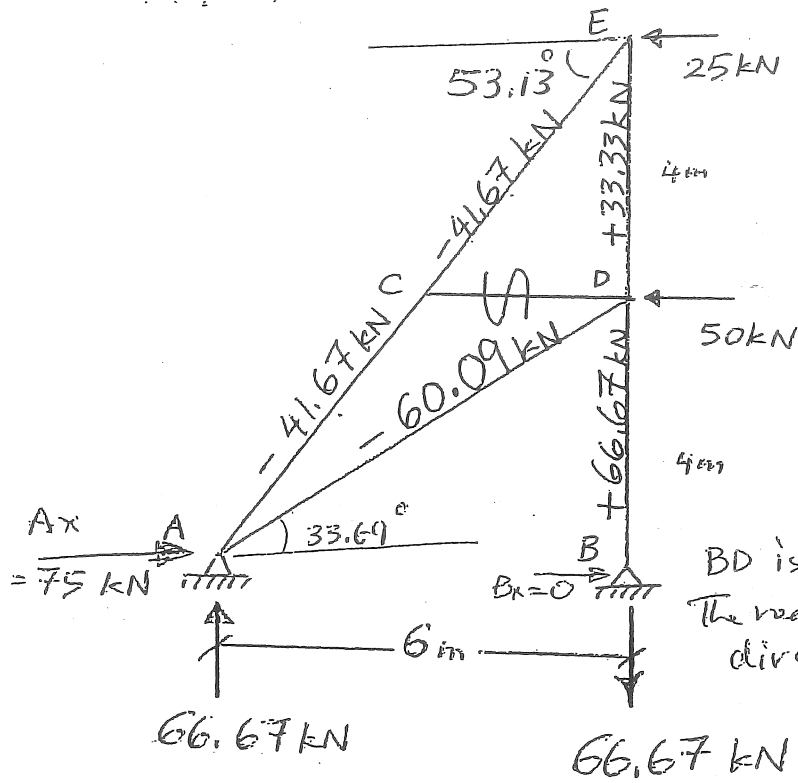
$$F_{JK} = 77.38 \text{ kN}$$

$$\sum F_x = 0 \rightarrow +$$

$$F_{IK} = 46.43 \text{ kN}$$

(3)

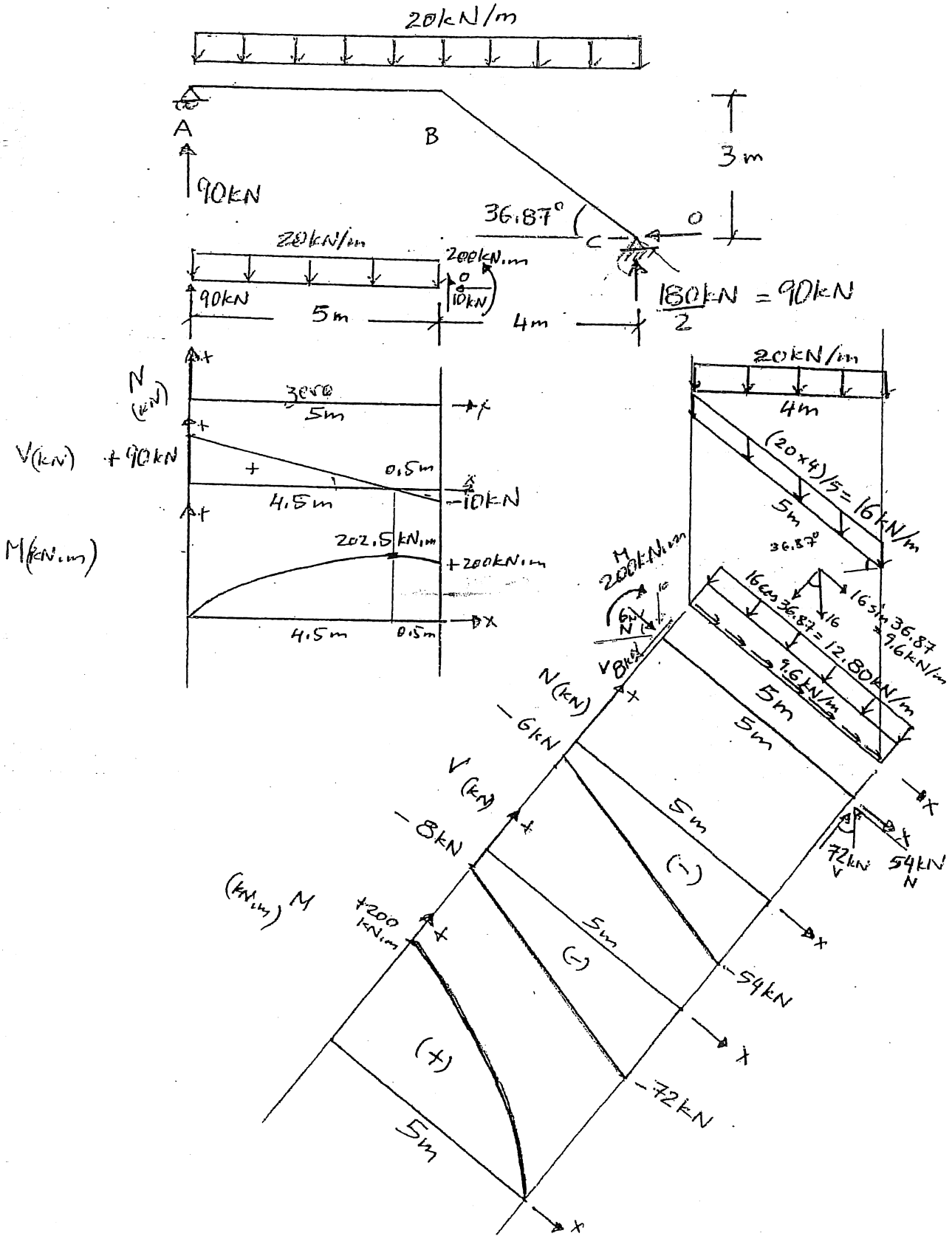
b) (4 points)



BD is a two-force member.
The reaction has a known direction.

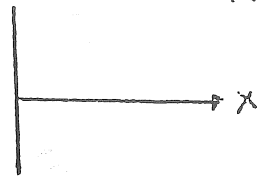
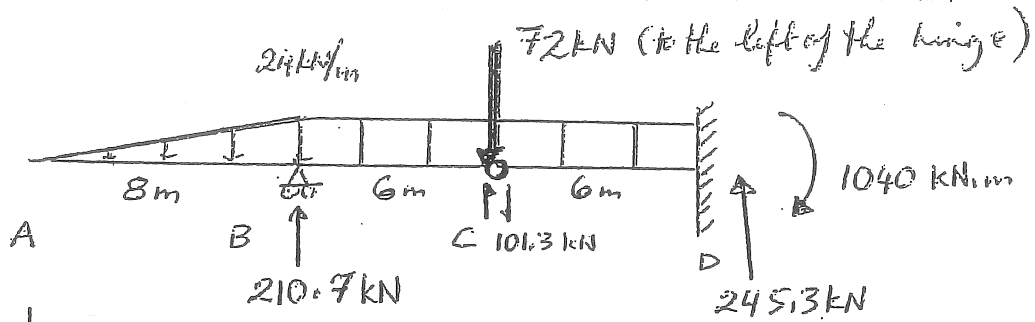
2. (10 points) Draw the axial, shear, and bending moment diagrams.

(5)

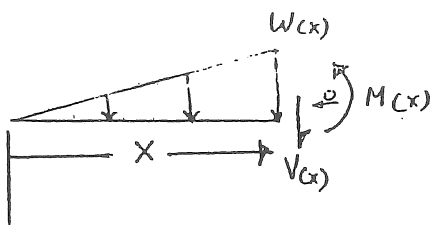


3. (12 points) Write the shear and moment functions using integration.

(7)



$0 \leq x < 8$



$W(x) = -3x$

$V(x) = -\frac{3x^2}{2} + C_1$

$V(0) = 0 \Rightarrow C_1 = 0$

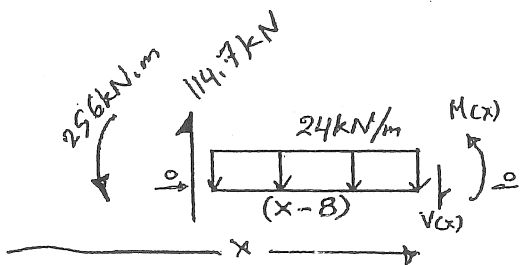
$\therefore V(x) = -\frac{3x^2}{2}$

$M(x) = -\frac{3x^3}{6} + C_2$

$M(0) = 0 \Rightarrow C_2 = 0$

$\therefore M(x) = -\frac{x^3}{2}$

$8 < x < 14$



$W(x) = -24$

$V(x) = -24x + C_3$

$V(8) = +114.7$

$114 = -24(8) + C_3 \Rightarrow C_3 = 306.7$

$\therefore V(x) = -24x + 306.7$

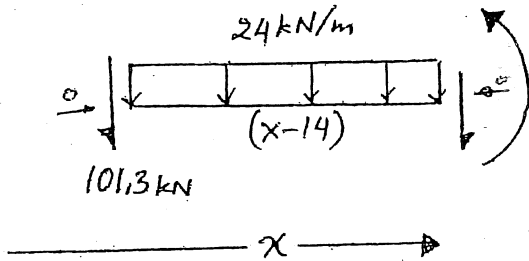
$M(x) = -\frac{24x^2}{2} + 306.7x + C_4$

$M(8) = -256$

$-256 = -12(8)^2 + 306.7(8) + C_4 \Rightarrow C_4 = -1941.6$

$\therefore M(x) = -12x^2 + 306.7x - 1941.6$

$$14 < x < 20$$



$$N(x) = -24 \quad (8)$$

$$V(x) = -24x + C_5$$

$$V(14) = -101.3$$

$$-101.3 = -24(14) + C_5 \Rightarrow C_5 = 234.7$$

$$\therefore V(x) = -24x + 234.7$$

$$M(x) = -\frac{24x^2}{2} + 234.7x + C_6$$

$$M(14) = 0$$

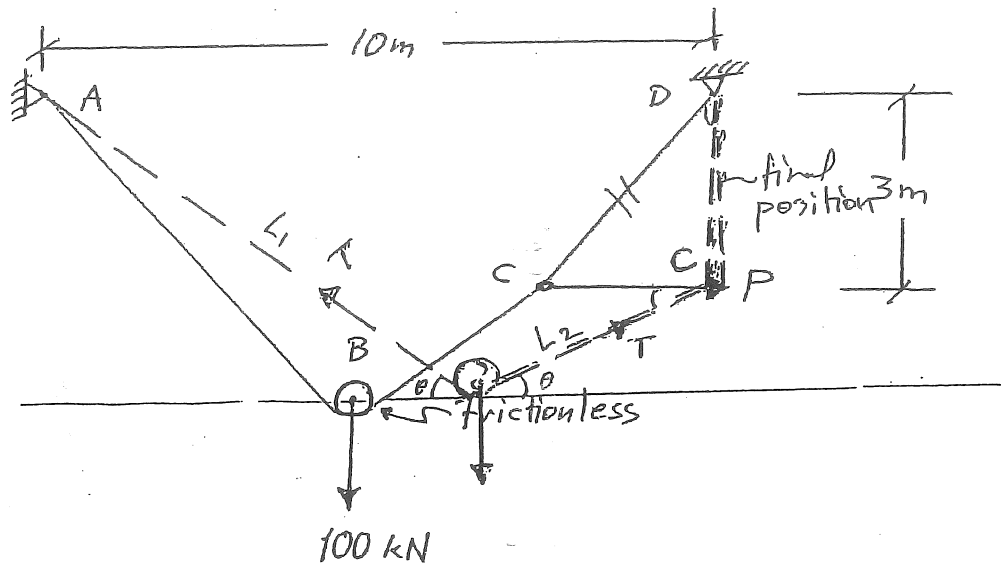
$$0 = -12(14)^2 + 234.7(14) + C_6$$

$$\Rightarrow C_6 = -934$$

$$\therefore M(x) = -12x^2 + 234.7x - 934$$

4. (6 points) Determine P such that segment CD remains vertical.
 Total length of the cable = 16 m. $CD = 3\text{ m}$

(9)



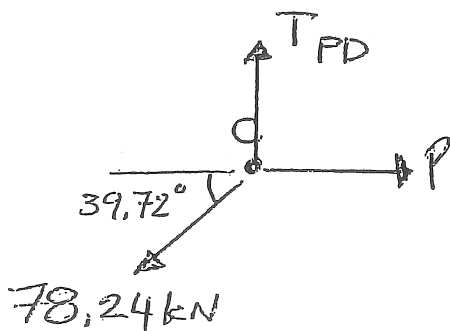
$$(L_1 + L_2) = 13\text{ m}$$

$$L_1 \cos \theta + L_2 \cos \theta = 10$$

$$\cos \theta (L_1 + L_2) = 10$$

$$\cos \theta (13) = 10 \implies \theta = 39.72^\circ$$

$$2T (\sin 39.72) = 100 \implies T = 78.24\text{ kN}$$



$$78.24 \cos 39.72 = P$$

$$\therefore P = 60.18\text{ kN}$$

S-M-A

Name: Saad Assi No.: 1100958 Sec.:

43/100

Birzeit University – faculty of engineering

Department of Civil Engineering

ENCE333 – First Hour Exam

ABU ASSI

Instructor: Ghada Karaki

First Semester 2012/2013

Q1) (a) Fill in the blanks

16

(20 points)

1. Principle of Superposition states

The total displacement or internal loading (stress) at a point on the structure subjected to several external loading can be determined by adding together the total displacement or internal loading (stress) caused by each of external loading acting separately.

2. And its requirements are we apply hooks law (in elastic state)
small def apply small deformation theorem.

3. wind load and earthquake are two examples of lateral loading and their distribution is along columns of the building height

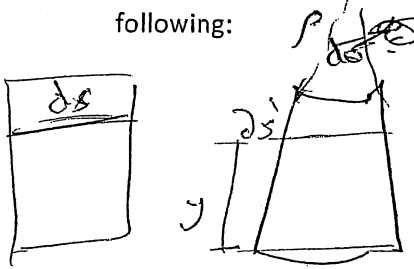
4. Live loads are gravity loads that vary in same direction and their minimum values can be found in national and international codes and are categorized depending on the shape of the structure.

5. A Truss can be defined as the structure composed from
of members the members are under compression (tension)

6. A frame can be described as the structure contain
and columns structural element like beams and columns and slab tension (tie)

7. Differential settlements have no effect on the internal stresses of statically indeterminate determinate systems. Whereas, they affect statically indeterminate systems and introduce additional load reaction when analyzed. internal stress and load cause compression and bending moment

8. Based on the beam-elastic theory, the relation between the internal moment in a beam to the displacement of its elastic curve (deflected shape) can be derived as the following:



by small deformation theorem

$$\epsilon = \frac{\Delta L}{L}$$
$$= \frac{(y + \delta s) - P \delta s}{P \delta s}$$

$$= \frac{y + \delta s - P \delta s - y \delta s}{P \delta s}$$

$$\epsilon = -\frac{y}{\delta s}$$

$$\epsilon = -\frac{y}{\delta s}$$

curvature

$$\sigma = E \epsilon$$

M u . E L y I

$$\sigma = E \epsilon$$
$$\frac{M y}{I} = E \frac{-y}{\delta s}$$

$$\frac{M}{I} y = E \left(\frac{y}{\rho} \right)$$

$$\frac{1}{\rho} = \frac{M}{EI}$$

$$\frac{d^2 y}{dx^2} = \frac{M}{EI}$$

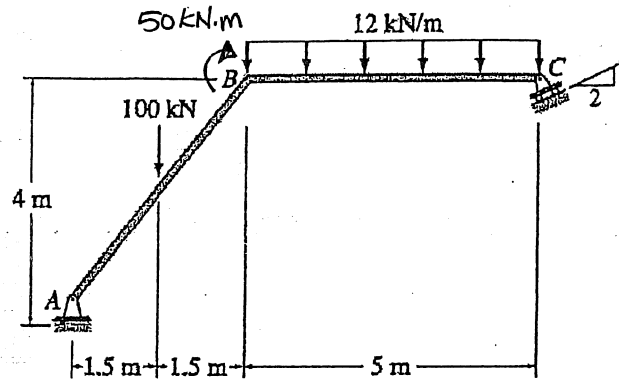
2nd derivative of deflection shape ✓

Q2)

(30 points)

For the determinate frame loaded as shown, draw indicating all key values

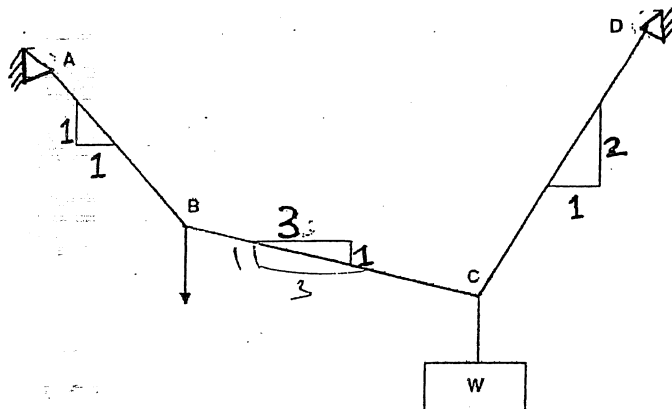
- The axial force diagram
- The shear force diagram
- The bending moment diagram



Q3)

A hoisting mechanism uses a cable system as shown below, If the load W is 20 kN, determine the effort or force at b required for the given arrangement

(20 points)



Q4)

(a) Use the conventional equilibrium approach to construct influence lines for the indicated response functions for the structure shown below

Required response functions

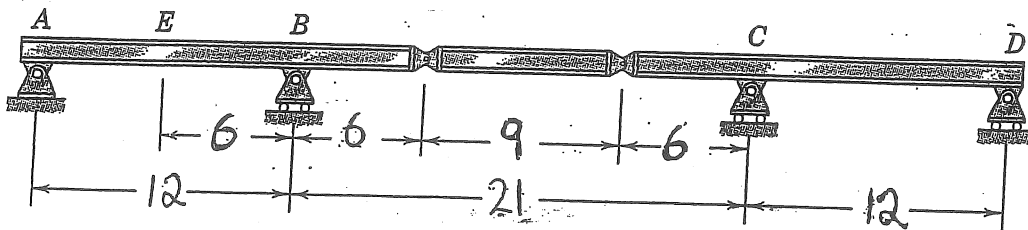
(24 points)

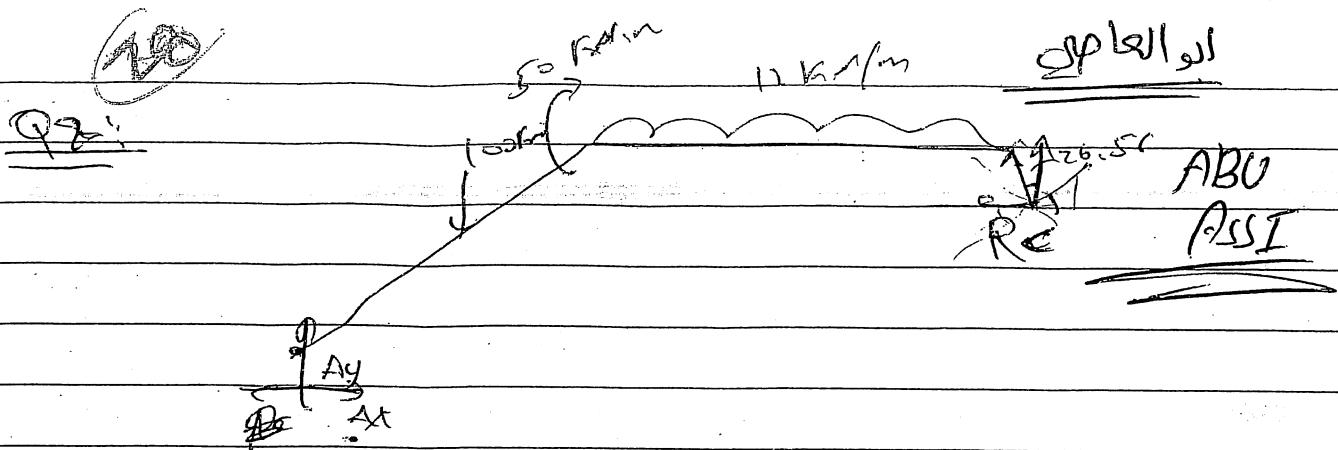
- All vertical reactions
- Moment at E
- Moment at C

(b) For the same structure and for the loading specified, determine the maximum negative moment at E

(6 points)

- Dead load: uniform of 2.5kN/m
- Live load: uniform of 4kN/m and a concentrated load of 30kN





$$\sum M_A = 0$$

$$+(R_c \cos 26.56 \times 8) + (R_c \sin 26.56 \times 4) + (12 \times 5 \times (\frac{5}{2} + 3)) - 50 - 100 \times 1.5 = 0$$

$$7.15 R_c + 1.728 R_c - 330 - 50 - 150 = 0$$

$$8.91 R_c = 530$$

$$R_c = 59.3 \text{ kN}$$

$$\sum F_y = 0$$

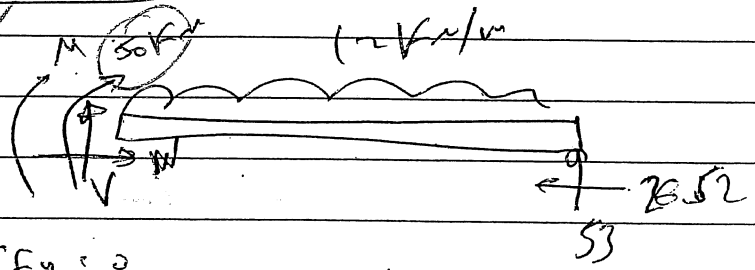
$$A_y - 100 - (12 \times 5) + R_c (\cos 26.56) = 0$$

$$A_y - 100 - 60 + 53 = 0$$

$$A_y = 107 \text{ kN}$$

$$\sum F_x = 0 \Rightarrow A_x = R_c \sin 26.56 = 5$$

$$A_x = 26.5 \text{ kN}$$



$$\sum F_y = 0$$

$$V + 53 - 60 = 0$$

$$V = 7 \text{ kN}$$

$$\sum F_x = 0$$

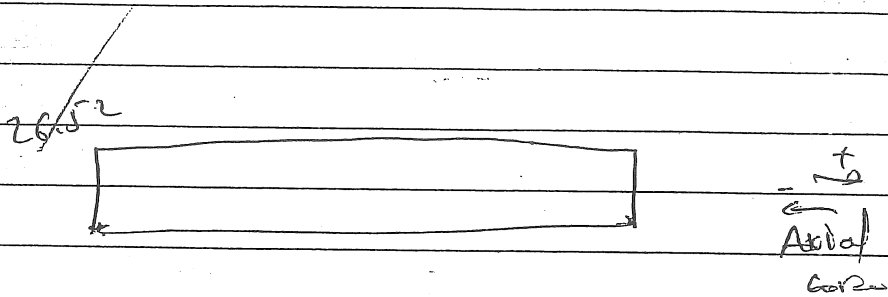
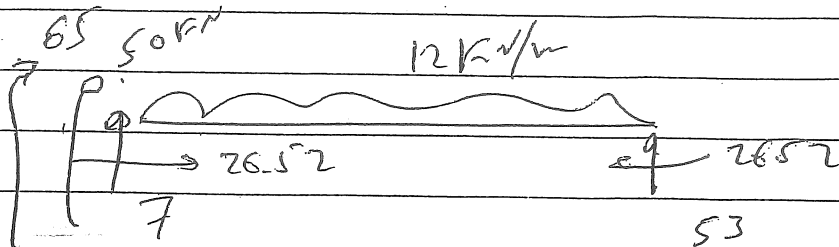
$$N = 26.52 \text{ kN}$$

$$\sum M_B = 0$$

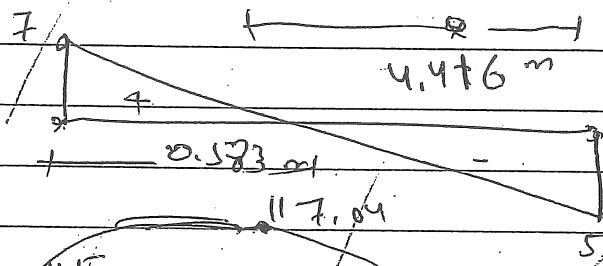
$$-50 - M + 53(5) + (12 \times 5) \frac{5}{2} = 0$$

$$-50 - M + 265 =$$

$$M = 65 \text{ KN}$$



$$\frac{50}{5} = \frac{7}{x}$$

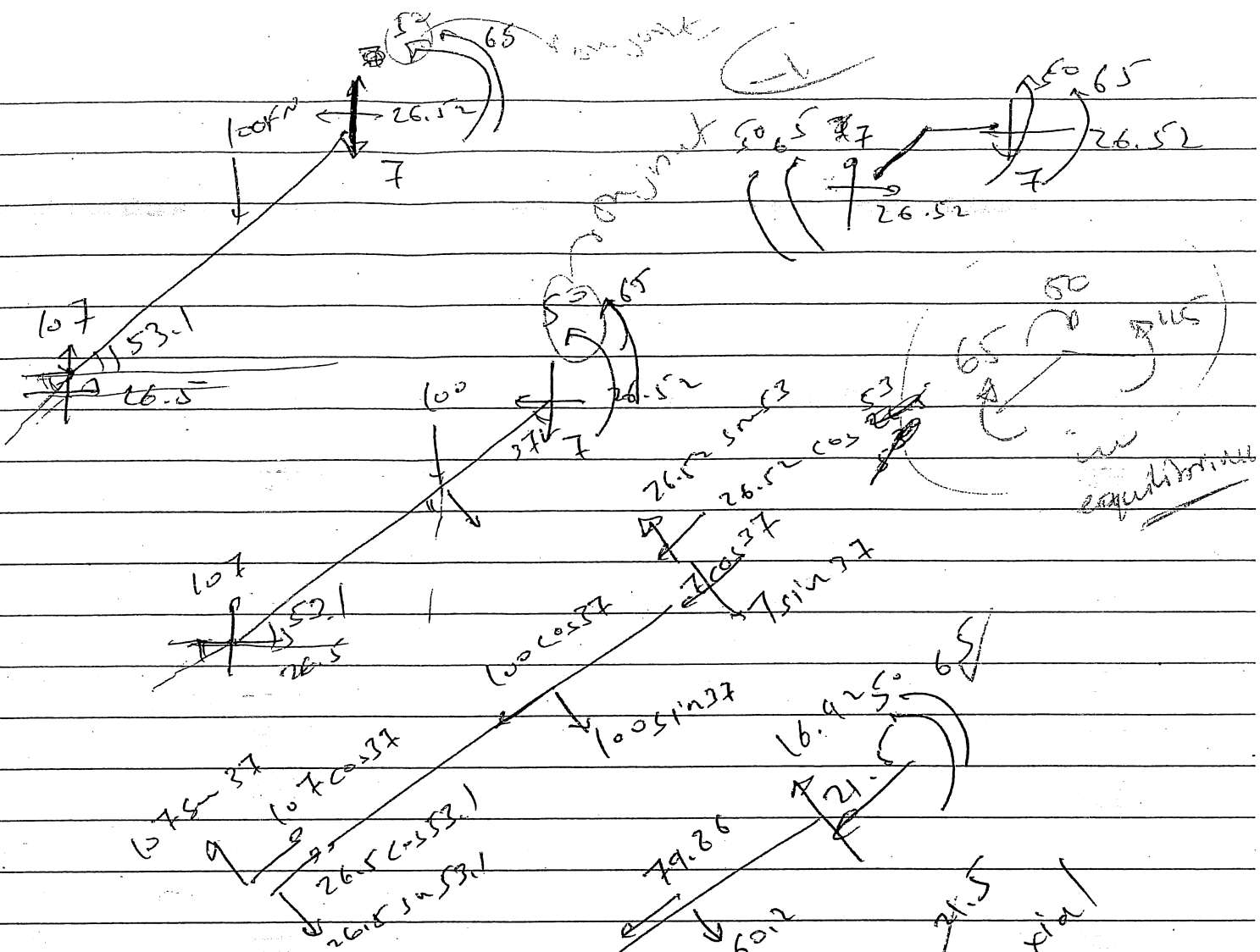


$$60 \times 35$$

$$x < 0.583$$

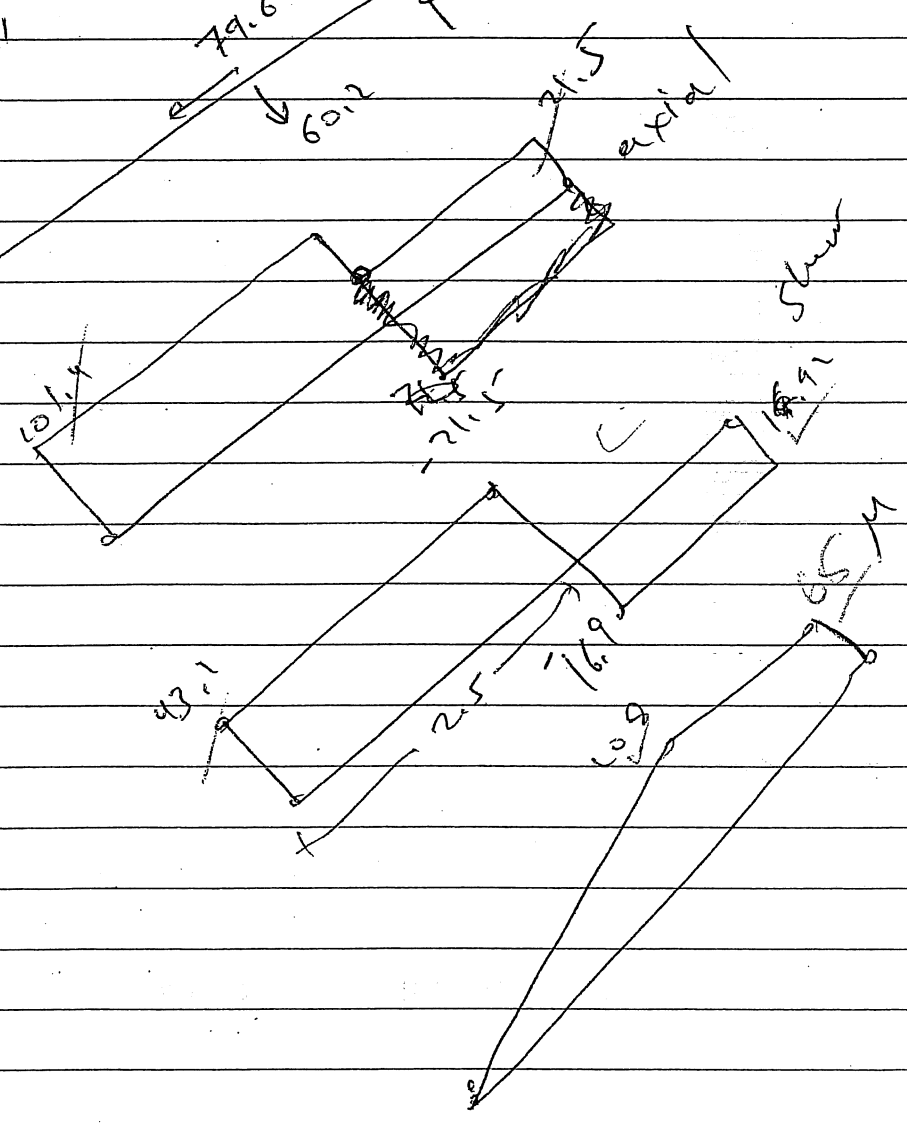
shear force

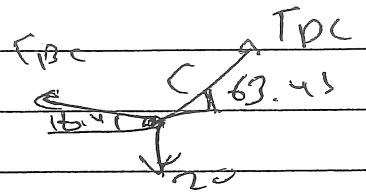
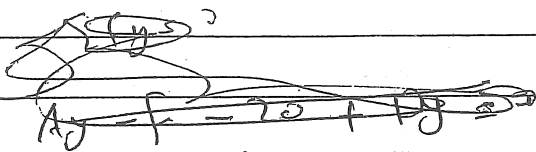
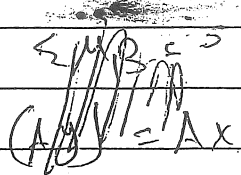
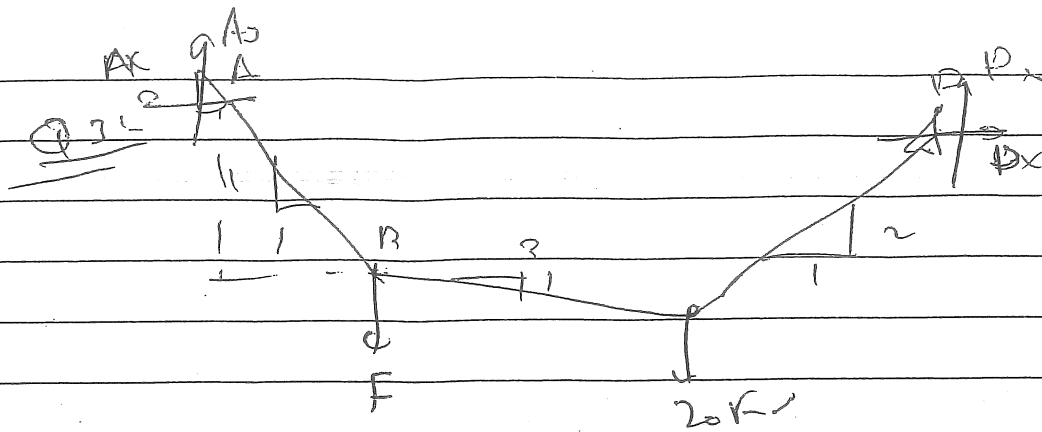
M diagram



64.4

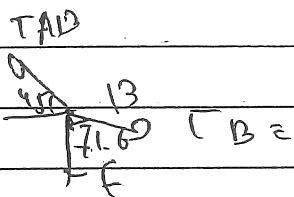
43.2
101.4





$$\sum F_y = 0$$

$$T_{DC} \sin 63.43 + T_{BC} \sin 37 = 20$$



$$0.894 T_{DC} + 0.315 T_{BC} = 20 \quad (1)$$

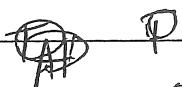
$$T_{DC} \cos 63.43 = T_{BC} \cos 37$$

$$0.747 T_{DC} = 0.95 T_{BC}$$

$$T_{AB} \cos 45$$

$$T_{BC} \sin 71.6 = T_{AB} \cos 45 \quad (1)$$

$$(2) \quad T_{DC} = T_{BC} (2.125)$$



$$(9.03) (0.95) = T_{AB} (0.707)$$

$$0.894 (T_{BC} (2.125)) + 0.315 T_{BC} = 20$$

$$T_{AB} = 12.13 \text{ kN}$$

$$1.9 T_{BC} + 0.315 T_{BC} = 20$$

$$T_{BC} = 9.03 \text{ kN}$$

$$\sum F_y = 0$$

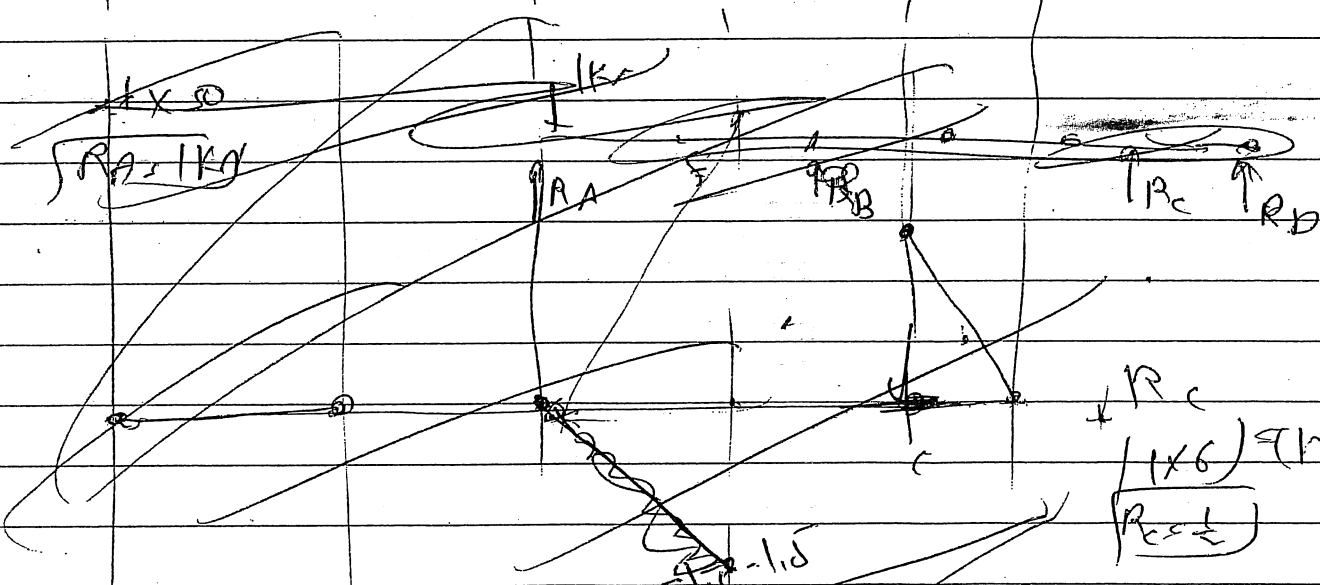
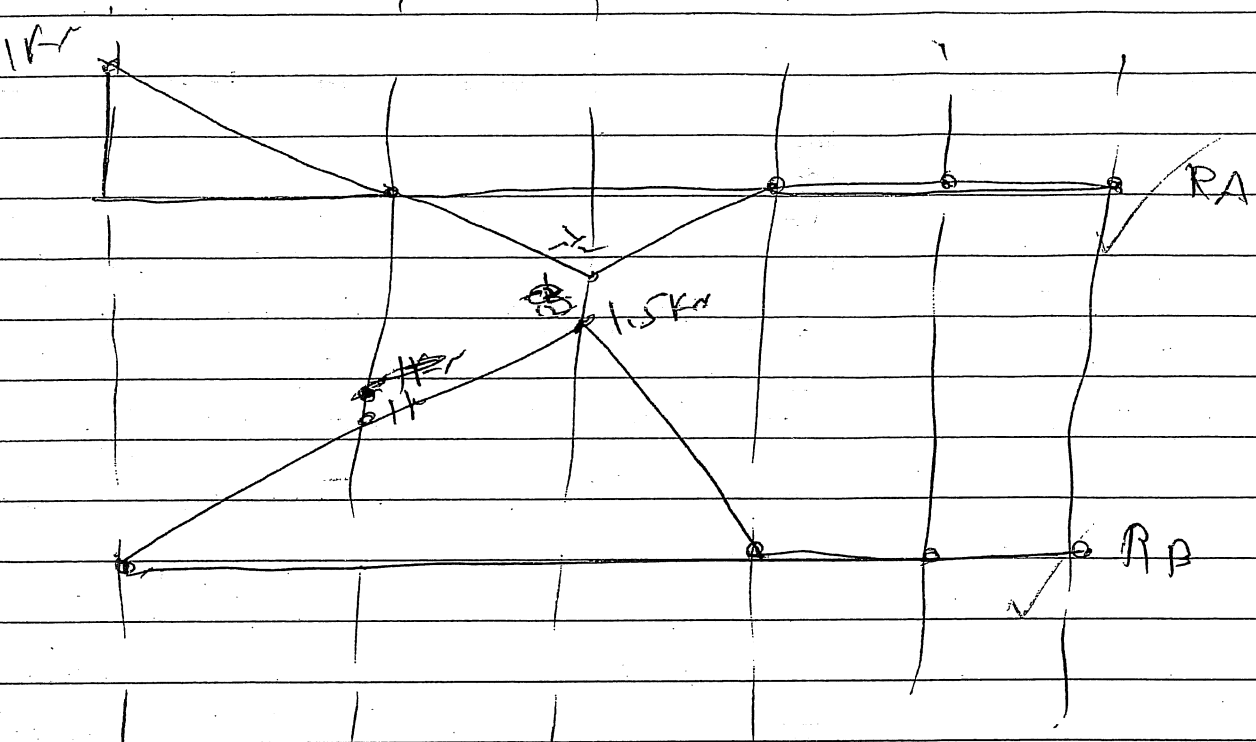
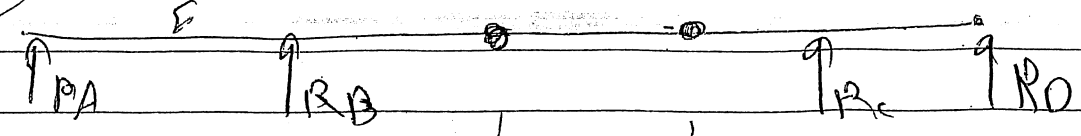
$$T_{DC} = 19.2 \text{ kN}$$

$$T_{AB} \sin 45 - T_{BC} \cos 71.6 - F = 0$$

$$12.13 \sin 45 - 9.03 \cos 71.6 - F = 0$$

$$8.577 - 2.85 - F = 0 \quad \boxed{F = 5.73 \text{ kN}}$$

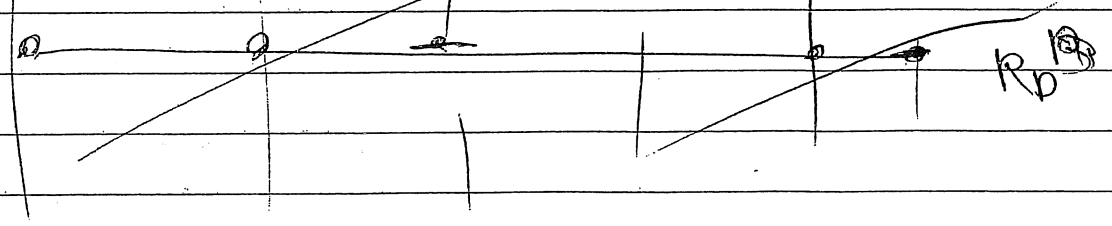
Q4 (2a)

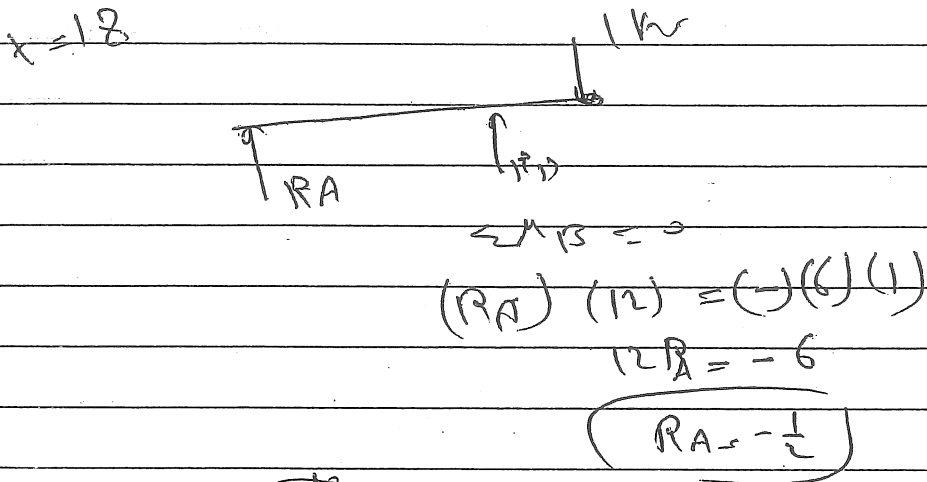
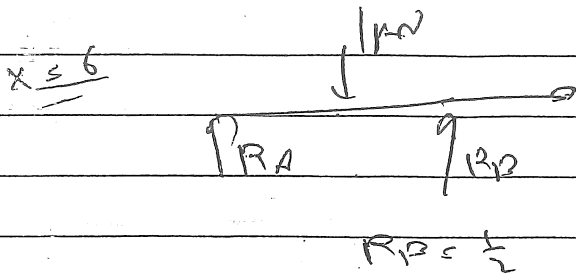
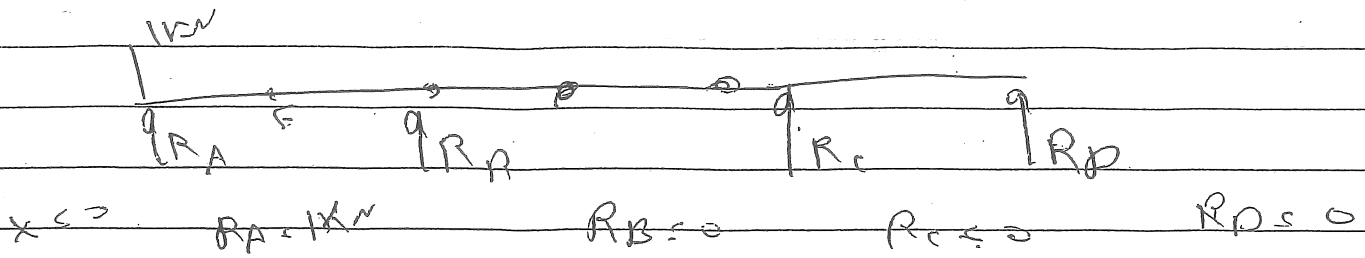
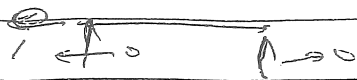


$$\frac{1 \times 6}{2} = 3$$

$$\frac{1 \times 1.5}{2} = 0.75$$

$$3 - 0.75 = 2.25$$



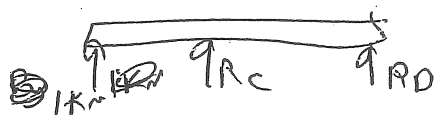
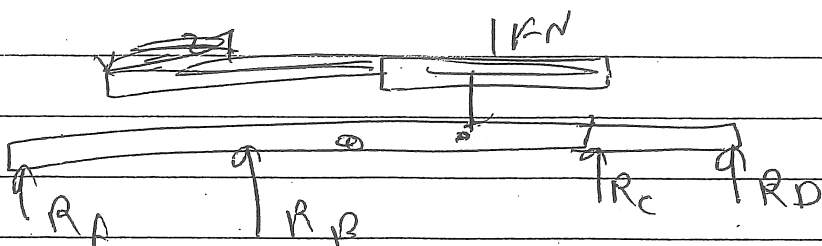
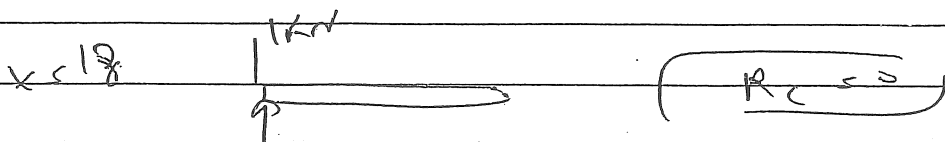
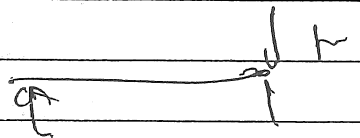


~~$R_B =$~~

$$\sum M_A = 0$$

$$(12)(R_B) = (18)(1)$$

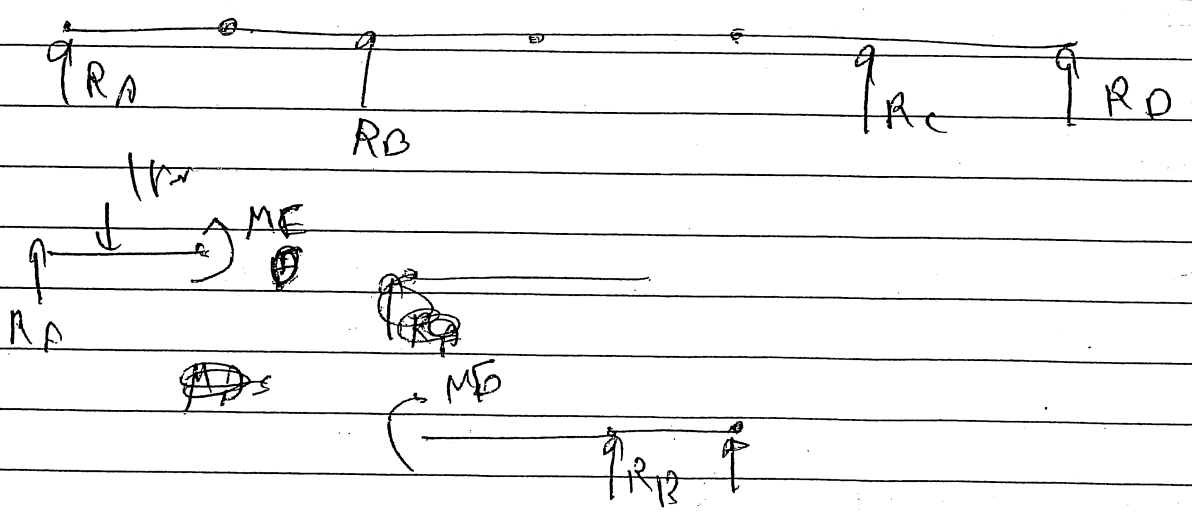
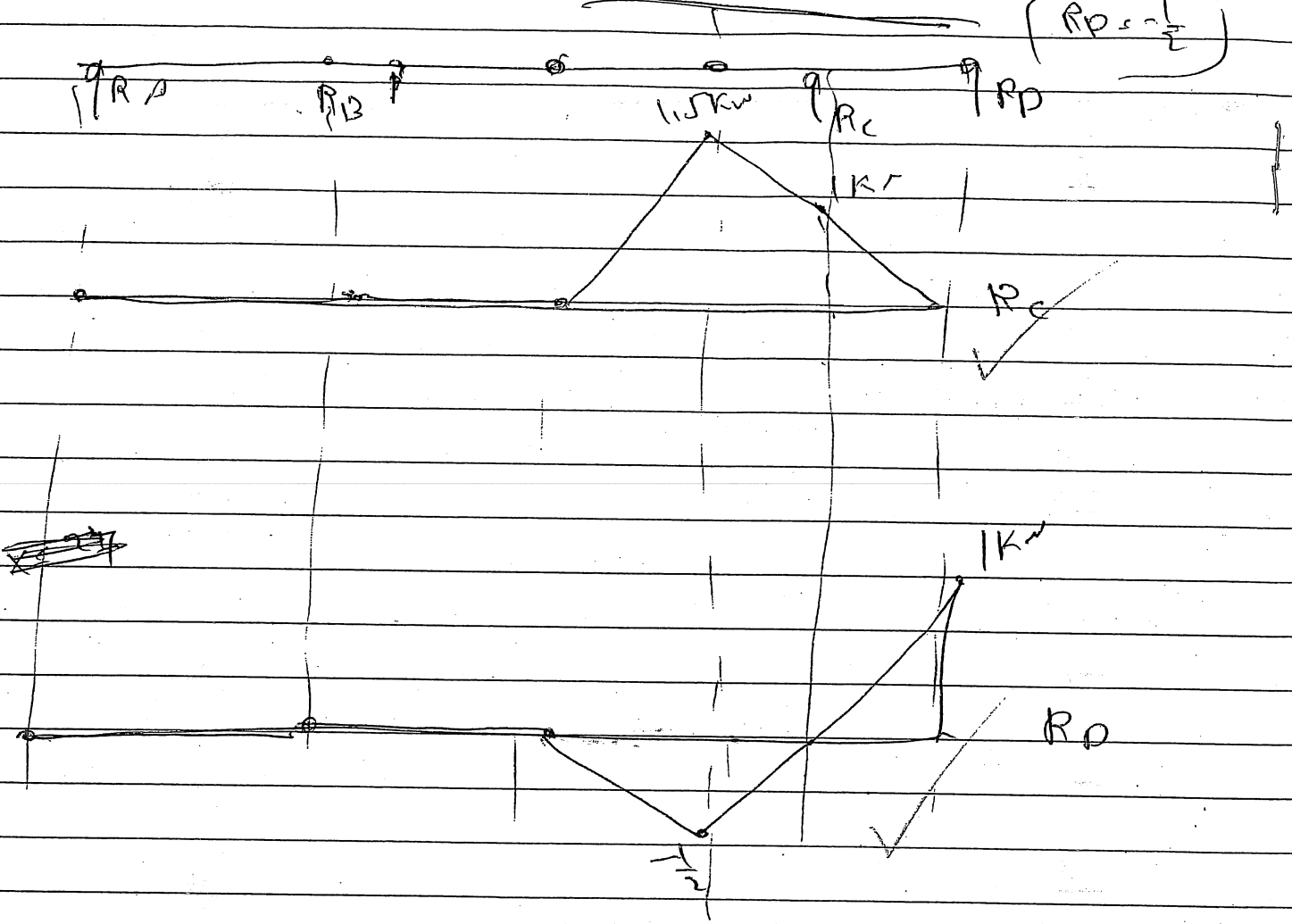
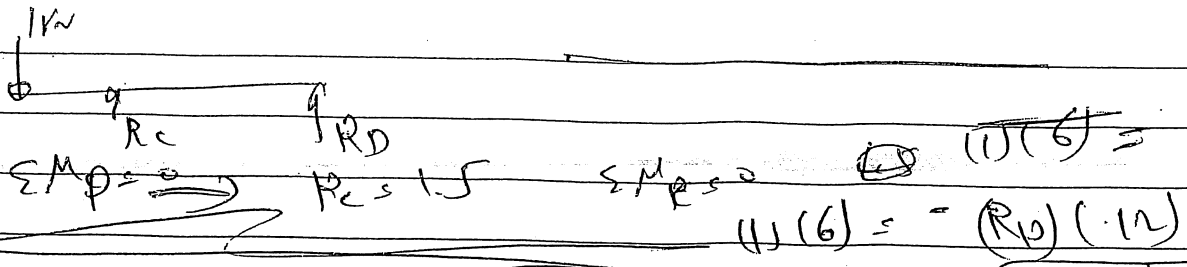
$$R_B = \frac{18}{12} = 1.5$$



$$\sum M_D = 0$$

$$(R_C)(12) = -1(18)$$

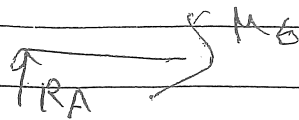
$$R_C = -1.5kN$$



~~$\Sigma M_D = 0$
 $(R_D)(6) = \frac{1}{2}$~~

6 x 45

$$(R_A)(6) = M_E$$

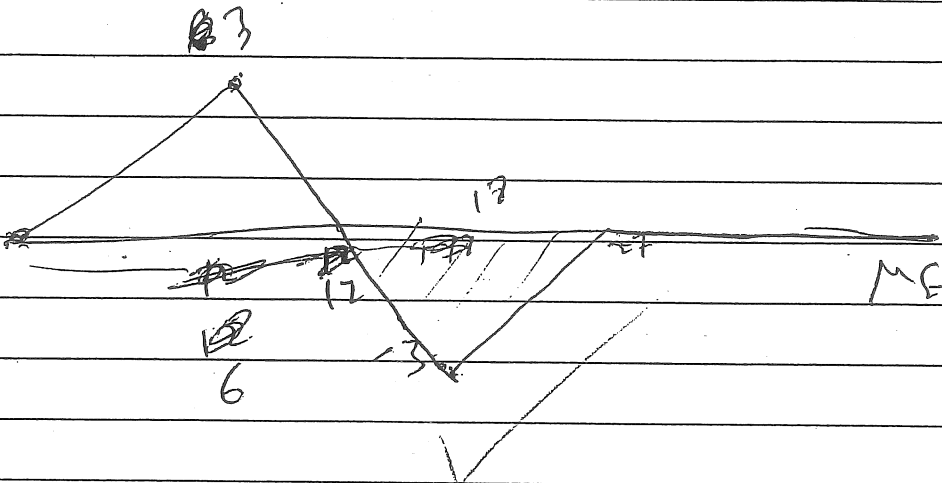


6 x 6

$$\sum M_E = 0$$

$$(R_B)(6) + (R_C)(27) + (R_D)(39) = M_E$$

$$6R_B + 27R_C + 39R_D = M_E$$



$$\sum M_E = 0 + 27 + 0$$

$$M_E = 0$$

~~P.L.~~

max ~~moment~~ moment at E = (2.5) x [(1/2)(8)(18) + (1/2)(-3)(15)]

$R_B = 1$

$$M_E = 6 \times 1 = 6 \text{ kNm}$$

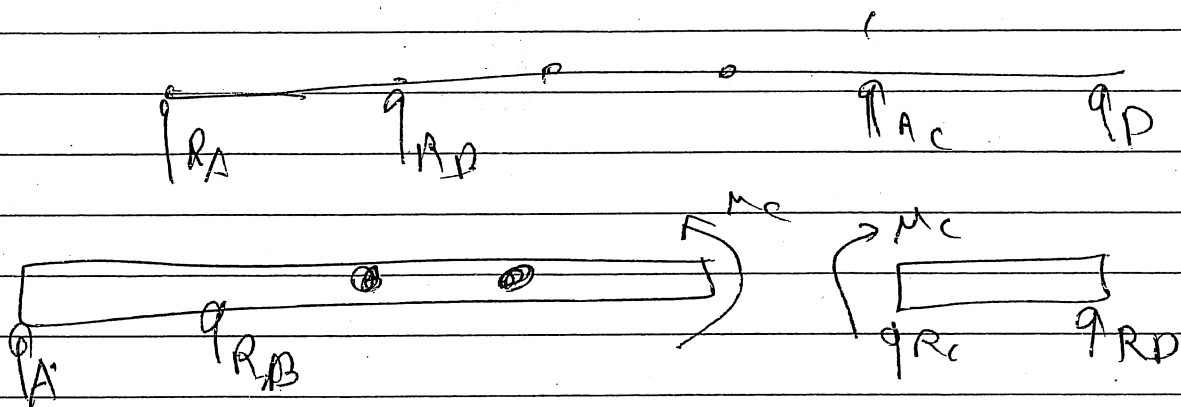
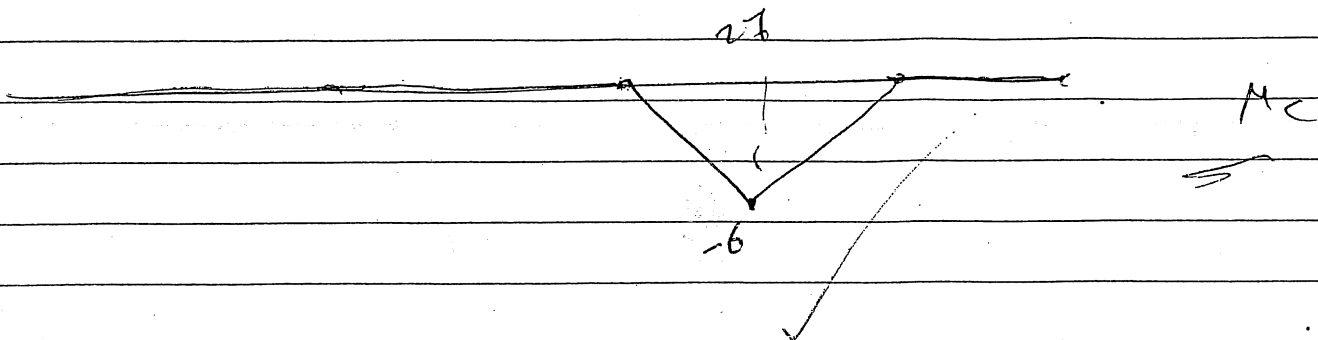
$$= 2.5 \times (27 - 22.5)$$

$$= 11.25 \text{ kNm}$$

max negative moment at E = (1/2)(-3)(15)(4) + (-3)(20)

$$= -90 + -180 = -180 \text{ kNm}$$

$$\sum -11.25 + -180 =$$



o x c 33

$$M_c = 12 P_D$$

33 c x c 45

$$M_c = (R_A)(33) + 21 P_B$$

$x = 18$ $M_c = 0$

$x = 27$

$$M_c = (12) \left(\frac{1}{2}\right) = -6$$